

Quiz 5

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §2.4 #75 (2 marks) Find the indicated limits, if they exist.

$$\begin{aligned} \lim_{x \rightarrow -\infty} \frac{3x^3 + x^2 + 1}{x^3 + 1} &= \lim_{x \rightarrow -\infty} \frac{(3x^3 + x^2 + 1) \left(\frac{1}{x^3}\right)}{(x^3 + 1) \left(\frac{1}{x^3}\right)} \\ &= \lim_{x \rightarrow -\infty} \frac{\left(3 + \frac{1}{x} + \frac{1}{x^3}\right)}{1 + \frac{1}{x^3}} = \frac{3}{1} = 3 \end{aligned}$$

Question 2. §2.5 #54 (3 marks) Find the values of x for which each function is continuous.

$$f(x) = \begin{cases} -2x+1 & \text{if } x < 0 \\ x^2+1 & \text{if } x \geq 0 \end{cases}$$

$$(2) \lim_{x \rightarrow 0^-} f(x) = 1 = \lim_{x \rightarrow 0^+} f(x)$$

$$(3) \lim_{x \rightarrow 0} f(x) = 1 = f(0)$$

$f(x)$ is continuous for $x < 0$
since $-2x+1$ is a polynomial.

$f(x)$ is continuous for $x > 0$
since x^2+1 is a polynomial.

\therefore continuous at $x=0$.

at $x=0$

(1) $f(0) = 1$, so defined

Question 3. §2.6 #26 Let

$$f(x) = \frac{1}{x-1}$$

a. (3 marks) Find the derivative f' of f .

b. (2 marks) Find an equation of the tangent line to the curve at the point $(-1, -\frac{1}{2})$.

$$\begin{aligned} a) \quad f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\frac{1}{x+h-1} - \frac{1}{x-1}}{h} \\ &= \lim_{h \rightarrow 0} \frac{\frac{x-1}{(x+h-1)(x-1)} - \frac{x+h-1}{(x-1)(x+h-1)}}{h} \\ &= \lim_{h \rightarrow 0} \frac{x-1 - x-h+1}{(x+h-1)(x-1)h} \\ &= \lim_{h \rightarrow 0} \frac{-h}{h(x+h-1)(x-1)} = \frac{-1}{(x-1)^2} \end{aligned}$$

$$b) \text{ Let's find the slope at } x=-1$$

$$m = f'(-1) = \frac{-1}{(-1-1)^2} = \frac{-1}{4}$$

$$\text{So } y = mx + b$$

$$y = \left(-\frac{1}{4}\right)x + b$$

$$-\frac{1}{2} = \left(-\frac{1}{4}\right)(-1) + b$$

$$-\frac{3}{4} = b$$

$$\therefore y = \frac{-1}{4}x - \frac{3}{4}$$

sub $(-1, -\frac{1}{2})$ to solve for b .