Dawson College: **Calculus I**: 201-NYA-05-S4: Winter 2012

Name: Y. Lamontagne
Student ID: ____________

Test 1

This test is graded out of 45 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(4 marks)* Simplify, expressing the answers with positive exponents only:

$$\frac{(a^{-2}b^3)^{-4}}{(a^{-3}b^2)^{-2}(ab)^{-4}} \cdot \left(\frac{c^0a^6}{b^4}\right)^{-\frac{1}{2}}$$

$$= \frac{(ab)^4(a^{-3}b^2)^2}{(a^{-2}b^3)^4} \cdot \left(\frac{b^4}{a^6}\right)^{1/2}$$

$$= \frac{a^4b^4a^{-6}b^4}{a^{-8}b^{12}} \cdot \frac{b^2}{a^3}$$

$$= \frac{a^{-2}b^{10}}{a^{-5} \cdot b^{12}} = \frac{a^{-2}a^5}{b^2} = \frac{a^3}{b^2}$$

$=$

Question 2. *(4 marks)* Simplify, expressing the answers with positive exponents only:

$$\frac{(4x^2-3)(3x^2+1)^{-\frac{1}{3}}-(3x^2+1)^{\frac{2}{3}}}{x+2}$$

$$= \frac{\frac{(4x^2-3)}{(3x^2+1)^{1/3}} - (3x^2+1)^{2/3}}{x+2}$$

$$= \frac{\frac{4x^2-3}{(3x^2+1)^{1/3}} - \frac{(3x^2+1)}{(3x^2+1)^{1/3}}}{x+2}$$

$$= \frac{\frac{x^2-4}{(3x^2+1)^{1/3}}}{x+2}$$

$$= \frac{x^2-4}{(x+2)(3x^2+1)^{1/3}}$$

$$= \frac{(x-2)\cancel{(x+2)}}{\cancel{(x+2)}(3x^2+1)^{1/3}}$$

$$= \frac{(x-2)}{(3x^2+1)^{1/3}}$$

Question 3. *(4 marks)* Given the supply and demand equations, where x represents the quantity demanded in units of a thousand and p the unit price in dollars, find the equilibrium quantity and the equilibrium price.

$$p = \frac{1}{x} \text{ and } p = x + 2 - \frac{62}{x}$$

$$\frac{1}{x} = x + 2 - \frac{62}{x}$$

$$\frac{x \cdot 1}{x} = x^2 + 2x - \frac{62x}{x}$$

$$0 = x^2 + 2x - 63$$

$$0 = (x+9)(x-7)$$

$x = \cancel{-9}$ not valid, $x = 7$

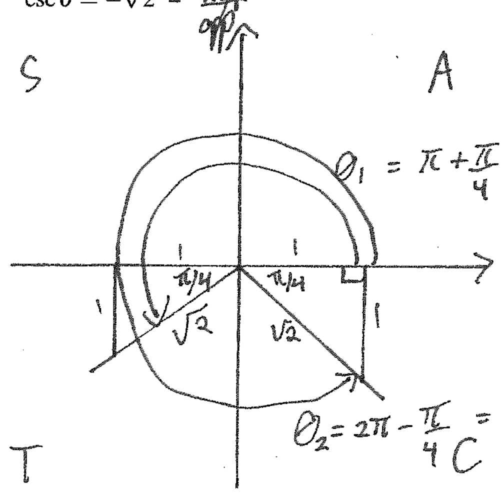
$\therefore p = \frac{1}{7}$.

$\therefore$ equilibrium at $x = 7000$ at a price of $\frac{1}{7}\$$

Question 4. *(4 marks)* Find all values of θ that satisfy the equation over the interval $[0, 2\pi]$

$\csc\theta = -\sqrt{2} = \frac{hyp}{opp}$

S A



$\theta_1 = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$

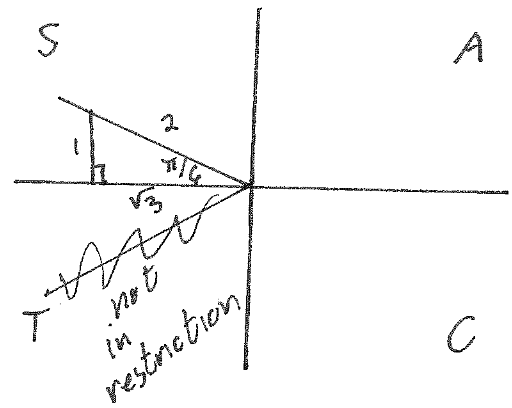
T C

$\theta_2 = 2\pi - \frac{\pi}{4} = \frac{7\pi}{4}$

Question 5. *(2 marks)* Find the exact value of:

$$\arccos\left(-\frac{\sqrt{3}}{2}\right) = \theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$$

$$\frac{\text{adj}}{\text{hyp}} = \frac{-\sqrt{3}}{2} = \cos\theta$$



Question 6. *(3 marks)* Find the indicated limit, if it exists

$$\lim_{x\to-5}\frac{\sqrt{14+x}-3}{x+5} \qquad \text{I.f. } \frac{0}{0}$$

$$= \lim_{x\to-5}\frac{\left(\sqrt{14+x}-3\right)}{x+5}\frac{\left(\sqrt{14+x}+3\right)}{\left(\sqrt{14+x}+3\right)}$$

$$= \lim_{x\to-5}\frac{14+x-9}{(x+5)\left(\sqrt{14+x}+3\right)}$$

$$= \lim_{x\to-5}\frac{\cancel{x+5}}{\cancel{(x+5)}\left(\sqrt{14+x}+3\right)}$$

$$= \frac{1}{\sqrt{14-5}+3}$$

$$= \frac{1}{3+3} = \frac{1}{6}$$

Question 7. *(3 marks)* Find the indicated limit, if it exists

$$\lim_{x\to-1}\frac{x^2-1}{x^4+2x^3+x^2} = \lim_{x\to-1}\frac{(x-1)(x+1)}{x^2(x^2+2x+1)}$$

$$= \lim_{x\to-1}\frac{(x-1)\cancel{(x+1)}}{x^2(x+1)\cancel{(x+1)}}$$

$$= \lim_{x\to-1}\frac{(x-1)}{x^2(x+1)}$$

D.N.E.

Question 8. *(3 marks)* Find the indicated limit, if it exists

$$\lim_{x\to 2} \frac{x-2}{-x^3+4x} \qquad \text{I.f. } \frac{0}{0}$$

$$= \lim_{x\to 2} \frac{(x-2)}{-x(x^2-4)}$$

$$= \lim_{x\to 2} \frac{\cancel{(x-2)}}{-x\cancel{(x-2)}(x+2)}$$

$$= \frac{1}{-2(2+2)} = -\frac{1}{8}$$

Question 9. *(2 marks)* Find the indicated limit, if it exists

$$\lim_{x\to -\infty} \frac{x^4-1}{x^3+x^2-2x} \longrightarrow -\infty \qquad \text{D.N.E.}$$

since degree of numerator is greater than degree of denominator.

Question 10. *(2 marks)* Find the indicated limit, if it exists

$$\lim_{x\to\infty} \frac{2x^4-x^3+x+1}{1-3x^4} = \lim_{x\to\infty} \frac{\frac{2x^4}{x^4} - \frac{x^3}{x^4} + \frac{x}{x^4} + \frac{1}{x^4}}{\frac{1}{x^4} - \frac{3x^4}{x^4}}$$

$$= \lim_{x\to\infty} \frac{2 - \cancel{\frac{1}{x}}^{\,0} + \cancel{\frac{1}{x^3}}^{\,0} + \cancel{\frac{1}{x^4}}^{\,0}}{\cancel{\frac{1}{x^4}}^{\,0} - 3}$$

$$= -\frac{2}{3}$$

Question 11. *(6 marks)* For which values of x is the following function continuous? Clearly explain your reasoning.

$$f(x) = \begin{cases} \frac{2x-1}{x^2-9} & \text{if } x < 4 \\ 2 & \text{if } x = 4 \\ x^{23}+x^3+x+2 & \text{if } x > 4 \end{cases}$$

for $x<4$: $f(x) = \frac{2x-1}{x^2-9}$ is continuous where $x^2-9 \neq 0$

$(x-3)(x+3) \neq 0$

$x \neq 3 \quad x \neq -3$

$\therefore$ for $x<4$, $f(x)$ is continuous everywhere except $x=3,-3$

for $x>4$: $f(x) = x^{23}+x^3+x+2$ is continuous everywhere since a polynomial.

Lets check continuity at $x=4$

① $f(4) = 2$, $\therefore$ defined

② $\lim_{x\to 4^-} f(x) = \lim_{x\to 4^-} \frac{2x-1}{x^2-9} = \frac{2(4)-1}{4^2-9} = \frac{7}{7} = 1$

$\lim_{x\to 4^+} f(x) = \lim_{x\to 4^+} x^{23}+x^3+x+2 = 4^{23}+4^3+4+2 \neq 1$

$\therefore \lim_{x\to 4} f(x)$ does not exists

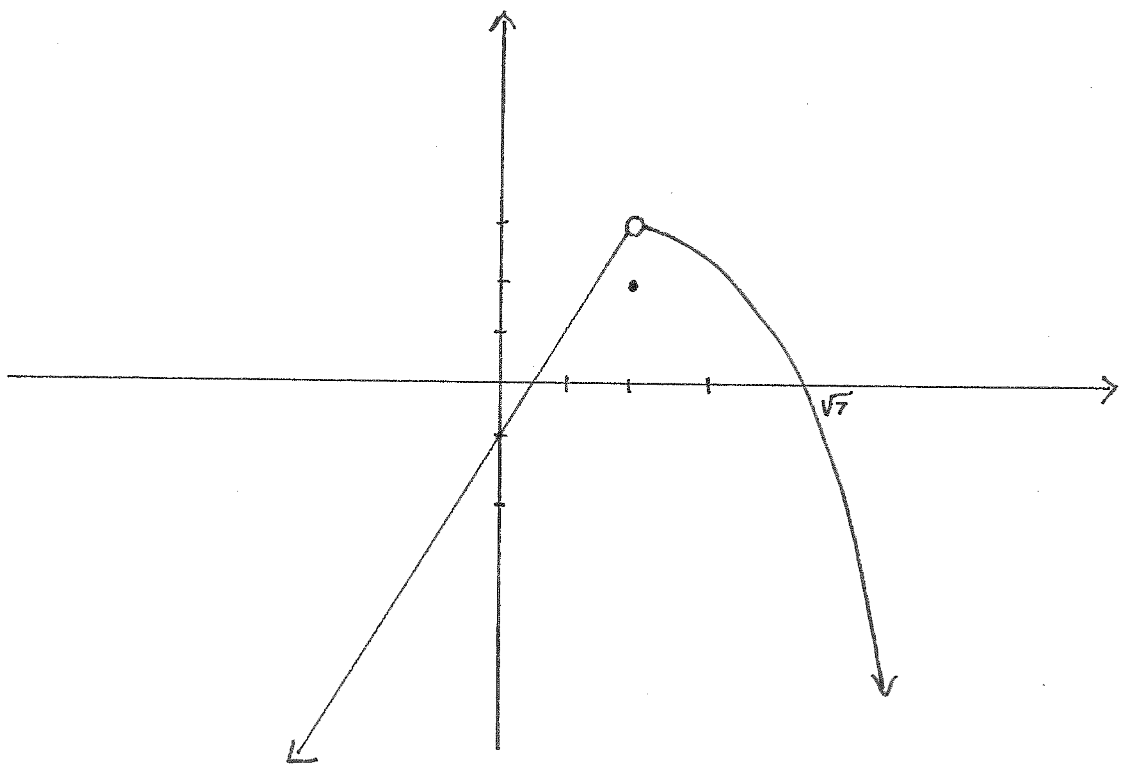
$\therefore$ not continuous at $x=4$

$\therefore f(x)$ is continuous on $(-\infty,-3)\cup(-3,3)\cup(3,4)\cup(4,\infty)$

$\equiv \mathbb{R} \setminus \{-3,3,4\}$

Question 12. *(4 marks)* Graph the following function and use the graph to determine where the function is continuous. If the function is discontinuous at a point $x = a$ state which condition of continuity fails.

$$f(x) = \begin{cases} 2x-1 & \text{if } x < 2 \\ 2 & \text{if } x = 2 \\ -x^2+7 & \text{if } x > 2 \end{cases}$$



Not continuous at $x=2$ since $\lim_{x \to 2} f(x) \neq f(2)$.

Question 13. Let

$$f(x) = x^2 - 6x + 1$$

a. *(3 marks)* Find the derivative f' of f using the limit definition of the derivative.

b. *(2 marks)* Find an equation of the tangent line to the curve at the point $x = -1$.

a) $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \frac{(x+h)^2 - 6(x+h) + 1 - [x^2 - 6x + 1]}{h}$

$= \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 \cancel{-6x} - 6h \cancel{+1} \cancel{-x^2} \cancel{+6x} \cancel{-1}}{h}$

$= \lim_{h \to 0} \frac{2xh + h^2 - 6h}{h}$

$= \lim_{h \to 0} \frac{\cancel{h}(2x + h - 6)}{h} = 2x - 6$

b) $\therefore$ slope of tangent at $x = -1$ is $m = f'(-1)$

$= 2(-1) - 6$

$= -8$

and a point that touches the tangent and curve is

$y = f(-1) = (-1)^2 - 6(-1) + 1 = 8$

So $(-1, 8)$

$\therefore$ $y = mx + b$ Solve for b by subs $(-1, 8)$

$y = -8x + b$

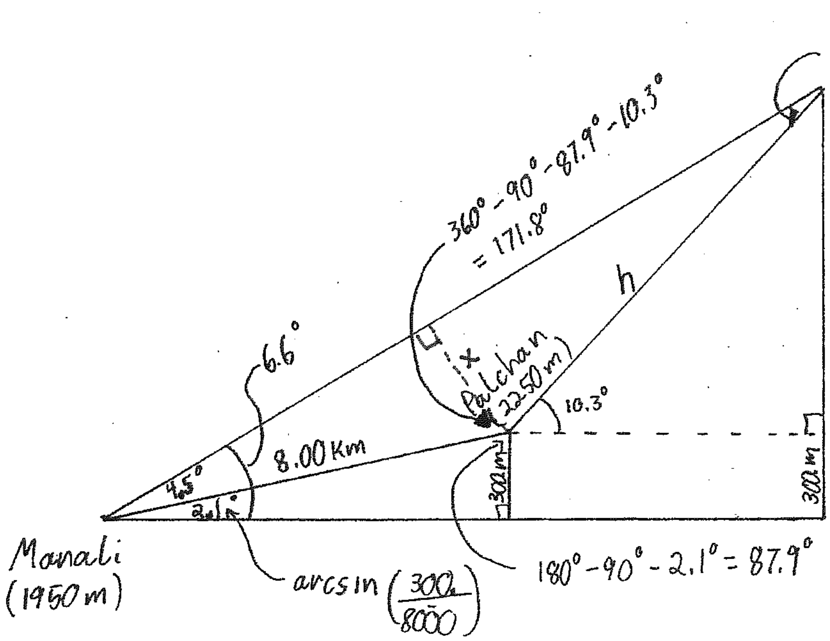
$8 = -8(-1) + b$

$0 = b$

$\therefore$ $y = -8x$ is the eqn. of the tangent of $f(x)$ at $x = -1$

Bonus Question. *(3 marks)*
In Yann's most recent trip, he must bike to the top of Rohtang Pass. Since Yann does not trust the author of his guide book, he wants to determine the height of the pass himself. Good thing he brought on his trip with him: a quadrant (to measure the *angle of elevation* from the horizon) and a GPS which displays his current elevation and total distance traveled.

The town of Manali is at an altitude of 1950. *m*. Before leaving Manali Yann measures the angle of elevation to the top of Rohtang Pass which reads 6.6°. He bikes on a straight road with constant slope, 8.00 *km* later he arrives in the town of Palchan at an altitude of 2250. *m* where he again measures the angle of elevation to the top of Rohtang Pass which reads 10.3°. What is the height of Rohtang Pass?



Lets determine h by using the Law of Sines

$$\boxed{\begin{aligned} \frac{h}{\sin 4.5°} &= \frac{8.00}{\sin 3.7°} \\ h &= \frac{8.00 \sin 4.5°}{\sin 3.7°} \\ &= 9.73 \text{ Km} \end{aligned}}$$

or

$\therefore$ $\sin 10.3° = \frac{0}{h}$

$0 = h \sin 10.3°$

$= 9.73 \sin 10.3°$

$= 1740$ m

$\therefore$ the height of Rohtang Pass is $1950 + 300 + 1740$
$= 3990$ m

$\sin 4.5° = \frac{x}{8.00}$

$x = 8.00 \sin 4.5°$

$\sin 3.7° = \frac{x}{h}$

$h = \frac{8.00 \sin 4.5°}{\sin 3.7°}$

$= 9.73$ Km.