Dawson College: **Calculus I**: 201-NYA-05-S4: Winter 2012

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Test 2

This test is graded out of 47 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(5 marks)* Find the derivative of the following function and simplify:

$$f(x) = \frac{(3x^2+2)^5}{(5x+1)^6}$$

$$f'(x) = \frac{5(3x^2+2)^4(6x)(5x+1)^6 - (3x^2+2)^5(6)(5x+1)^5(5)}{(5x+1)^{12}}$$

$$= \frac{30(3x^2+2)^4\cancel{(5x+1)^5}\left[x(5x+1) - (3x^2+2)\right]}{(5x+1)^{\cancel{12}\,7}}$$

$$= \frac{30(3x^2+2)^4(5x^2+x-3x^2+2)}{(5x+1)^7}$$

$$= \frac{30(3x^2+2)^4(2x^2+x+2)}{(5x+1)^7}$$

Question 2. *(5 marks)* Find the derivative of the following function:

$$g(t) = t^3 + \frac{1}{\sqrt{t}} + 2\sin t + 3\csc t + 4\arctan t$$

$$g'(t) = 3t^2 - \frac{1}{2}t^{-3/2} + 2\cos t - 3\csc t\cot t + \frac{4}{t^2+1}$$

Question 3. *(4 marks)* Find the derivative of the following function:

$$h(z) = \sqrt{\operatorname{arcsec}(z^2+1)} = (\operatorname{arcsec}(z^2+1))^{1/2}$$

$$h'(z) = \frac{1}{2}(\operatorname{arcsec}(z^2+1))^{-1/2} \frac{1}{(z^2+1)\sqrt{(z^2+1)^2-1}} \cdot 2z$$

Question 4. *(4 marks)* Find the derivative of the following function:

$$f(x) = (3x^3+x)^{3/2}\sqrt{2x-1} = (3x^3+x)^{3/2}(2x-1)^{1/2}$$

$$f'(x) = \frac{3}{2}(3x^3+x)^{1/2} \cdot (9x+1)\sqrt{2x-1} + (3x^3+x)^{3/2}\frac{1}{2}(2x-1)^{-1/2} \cdot 2$$

Question 5. The yearly demand for the Super Cool Fixed Gear bicycle is

$$p = 1000 - 0.04x \quad (0 \leq x \leq 20000)$$

where p denotes the wholesale unit price in dollars and x denotes the quantity demanded. The weekly total cost function associated with manufacturing the Super Cool Fixed Gear is given by

$$C(x) = 0.000003x^3 - 0.02x^2 + 300x + 70000$$

where C(x) denotes the total cost incured in producing x Super Cool Fixed Gear.

a. *(2 marks)* Find the revenue function R and the profit function P.

b. *(3 marks)* Find the marginal cost function C', the marginal revenue function R' and the marginal profit function P'.

c. *(2 marks)* Compute $P'(2000)$ and interpret your results.

a) $R(x) = xp$
$= x(1000 - 0.04x)$
$= 1000x - 0.04x^2$

$P(x) = R(x) - C(x)$
$= 1000x - 0.04x^2 - [0.000003x^3 - 0.02x^2 + 300x + 70\,000]$
$= -0.000003x^3 - 0.02x^2 + 700x - 70000$

b) $C'(x) = 0.000009x^2 - 0.04x + 300$
$R'(x) = 1000 - 0.08x$
$P'(x) = -0.000009x^2 - 0.04x + 700$

c) $P'(2000) = -0.000009(2000)^2 - 0.04(2000) + 700 = 584$

The additional profit by selling the 2001st bike will be about \$584.

Question 6. *(6 marks)* Find the third derivative of the following function. Simplify your final answer.

$$f(x) = \frac{x^2+1}{(2x+1)^2}$$

$$f'(x) = \frac{2x(2x+1)^2 - (x^2+1)2(2x+1)\cdot 2}{(2x+1)^4}$$

$$= \frac{2\cancel{(2x+1)}\left[x(2x+1) - 2(x^2+1)\right]}{(2x+1)^{\cancel{4}\,3}}$$

$$= \frac{2\left[2x^2 + x - 2x^2 - 2\right]}{(2x+1)^3}$$

$$= \frac{2(x-2)}{(2x+1)^3}$$

$$f''(x) = \frac{2(2x+1)^3 - 3(2x+1)^2(2)\,2(x-2)}{(2x+1)^6}$$

$$= \frac{2(2x+1)^{\cancel{2}}\left[(2x+1) - 6(x-2)\right]}{(2x+1)^{\cancel{6}\,4}}$$

$$= \frac{2\left[2x+1-6x+12\right]}{(2x+1)^4}$$

$$= \frac{-8x+26}{(2x+1)^4}$$

$$f'''(x) = \frac{(-8)(2x+1)^4 - (-8x+26)\,4(2x+1)^3(2)}{(2x+1)^8}$$

$$= \frac{(-8)(2x+1)^{\cancel{3}}\left[(2x+1) + (-8x+26)\right]}{(2x+1)^{\cancel{8}\,5}}$$

$$= \frac{(-8)\left[-6x+27\right]}{(2x+1)^5}$$

$$= \frac{24(2x-9)}{(2x+1)^5}$$

Question 7. Using the demand function:

$$p = 1000 - 0.04x$$

a. *(3 marks)* Calculate the elasticity of demand function $E(p)$.

b. *(2 marks)* Calculate the elasticity of demand when the price is set at \$750. Interpret the result.

c. *(1 mark)* At what price is the demand unitary?

a) $p = 1000 - 0.04x$

$0.04x = 1000 - p$

$x = 25000 - 25p$

$\therefore \quad R(p) = 25\,000 - 25p$

$\therefore \quad R'(p) = -25$

$$E(p) = \frac{-pR'(p)}{R(p)} = \frac{-p(-25)}{25000 - 25p} = \frac{25p}{25000 - 25p}$$

b) $E(750) = \dfrac{25(750)}{25000 - 25(750)}$

$= 3$

$\therefore$ if the price is raised 1% above \$750 then the demand decrease by 3%.

c) The demand is unitary if $E(p) = 1$

$$1 = \frac{25p}{25000 - 25p}$$

$25000 - 25p = 25p$

$25\,000 = 50p$

$p = 500\$$

$\therefore$ at $p = 500\$$ the demand is unitary.

Question 8. Given:

$6+\sqrt{2} - 2xy = \sqrt{y+1}$

a. *(5 marks)* Find $\frac{dy}{dx}$.

b. *(2 marks)* Find the equation of the tangent to the graph of the given relation at $(1,3)$

a) $\frac{d}{dx}\left[6+\sqrt{2}-2xy\right] = \frac{d}{dx}\left[\sqrt{y+1}\right]$

$-2y - 2xy' = \frac{1}{2}(y+1)^{-\frac{1}{2}}y'$

$-2y = \frac{1}{2}(y+1)^{-\frac{1}{2}}y' + 2xy'$

$-2y = y'\left(\frac{1}{2}(y+1)^{-1/2} + 2x\right)$

$y' = \dfrac{-2y}{\left(\frac{1}{2}(y+1)^{-1/2} + 2x\right)}$

b) $\therefore$ slope of tangent

$m = \dfrac{-2(3)}{\left(\frac{1}{2}(3+1)^{-1/2} + 2(1)\right)}$

$= \dfrac{-6}{\left(\frac{1}{2\sqrt{4}} + 2\right)} = \dfrac{-6}{\left(\frac{1}{4}+2\right)} = \dfrac{-6}{\frac{9}{4}} = \dfrac{-24}{9} = \dfrac{-8}{3}$

$\therefore\ y = mx + b$

$3 = \frac{-8}{3}(1) + b$

$b = \frac{17}{3}$

$\therefore$ tangent is $y = \frac{-8}{3}x + \frac{17}{3}$

Question 9. *(5 marks)* Find the equation of the tangent to the function $f(x) = \tan 2x$ at $x = \frac{\pi}{8}$.

$$f'(x) = \sec^2(2x) \cdot 2$$

$\therefore$ slope of tangent at $x = \frac{\pi}{8}$ is $m = f'(\frac{\pi}{8}) = \sec^2(2\frac{\pi}{8}) \cdot 2$

$$= \sec^2(\frac{\pi}{4})\ 2$$

$$= (\sqrt{2})^2 \cdot 2$$

$$= 4$$

and y-component of intersection between $f(x)$ and the tangent $y = f(\frac{\pi}{8}) = \tan 2(\frac{\pi}{8}) = \tan\frac{\pi}{4} = 1$

$\therefore (\frac{\pi}{8}, 1)$

$\therefore$

$$y = mx + b$$

$$y = 4x + b$$

$$1 = 4(\frac{\pi}{8}) + b$$

$$1 - \frac{\pi}{2} = b$$

$\therefore$ tangent $y = 4x + 1 - \frac{\pi}{2}$

Bonus Question. *(5 marks)*
Find the derivative of the following function but do *not* simplify.

$$f(x) = \cot\left(\sqrt{\frac{\arcsin(\tan x + x^2) + x}{\operatorname{arccot}(\sec x + e) - x}} + \pi\right)$$

$$f'(x) = -\csc^2\left(\sqrt{\frac{\arcsin(\tan x + x^2) + x}{\operatorname{arccot}(\sec x + e) - x}} + \pi\right) \cdot \frac{1}{2} \cdot \left(\frac{\arcsin(\tan x + x^2) + x}{\operatorname{arccot}(\sec x + e) - x}\right)^{-\frac{1}{2}} \cdot$$

$$\frac{\left[\left[\frac{1}{\sqrt{1-(\tan x + x^2)^2}} \cdot (\sec^2 x + 2x) + 1\right](\operatorname{arccot}(\sec x + e) - x) - (\arcsin(\tan x + x^2) + x)\left[\frac{-1}{(\sec x + e)^2 + 1} \cdot (\sec x \tan x) - 1\right]\right]}{(\operatorname{arccot}(\sec x + e) - x)^2}$$