Dawson College: Calculus II (SCIENCE): 201-NYB-05-S2: Winter 2012

Name: Y. Lamontagne
Student ID:

Quiz 10

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(2 marks)* §8.2 #15 Determine whether the series is convergent or divergent. If it is convergent, find its sum.

$$\sum_{n=1}^{\infty} \underbrace{\sqrt[n]{2}}_{a_n} \qquad \lim_{n\to\infty} a_n = \lim_{n\to\infty} \sqrt[n]{2} = \lim_{n\to\infty} 2^{1/n} = 2^0 = 1 \neq 0$$

$\therefore$ diverges by n^{th} term divergence test

Question 2. *(4 marks)* §8.2 #20 Determine whether the series is convergent or divergent by expressing S_n as a telescoping sum. If it is convergent find its sum.

$$\sum_{n=1}^{\infty} \frac{2}{n^2+4n+3} = \sum_{n=1}^{\infty} \frac{2}{(n+3)(n+1)}$$

$$\frac{2}{(n+3)(n+1)} = \frac{A}{(n+3)} + \frac{B}{(n+1)}$$

$$2 = A(n+1) + B(n+3)$$

Let $n=-1$: $B=1$

Let $n=-3$: $A=-1$

$$= \sum_{n=1}^{\infty} \left[\frac{1}{n+1} - \frac{1}{n+3}\right]$$

$$S_n = a_1 + a_2 + a_3 + a_4 + a_5 + \dots + a_{n-4} + a_{n-3} + a_{n-2} + a_{n-1} + a_n$$

$$= \left[\frac{1}{2} - \cancel{\frac{1}{4}}\right] + \left[\frac{1}{3} - \cancel{\frac{1}{5}}\right] + \left[\cancel{\frac{1}{4}} - \cancel{\frac{1}{6}}\right] + \left[\cancel{\frac{1}{5}} - \frac{1}{7}\right] + \left[\cancel{\frac{1}{6}} - \frac{1}{8}\right] + \dots$$

$$+ \left[\cancel{\frac{1}{n-3}} - \cancel{\frac{1}{n-1}}\right] + \left[\cancel{\frac{1}{n-2}} - \cancel{\frac{1}{n}}\right] + \left[\cancel{\frac{1}{n-1}} - \cancel{\frac{1}{n+1}}\right] + \left[\cancel{\frac{1}{n}} - \frac{1}{n+2}\right]$$

$$+ \left[\cancel{\frac{1}{n+1}} - \frac{1}{n+3}\right] = \frac{1}{2} + \frac{1}{3} - \frac{1}{n+2} - \frac{1}{n+3}$$

$$S = \lim_{n\to\infty} S_n = \lim_{n\to\infty}\left[\frac{1}{2} + \frac{1}{3} - \underbrace{\cancel{\frac{1}{n+2}}}_{\to 0} - \underbrace{\cancel{\frac{1}{n+3}}}_{\to 0}\right] = \frac{5}{6}$$

Question 3. *(4 marks)* §8.1 #27 Find the values of x for which the series converges. Find the sum of the series for those values of x.

$$\sum_{n=1}^{\infty} \frac{x^n}{3^n} = \sum_{n=1}^{\infty} \left(\frac{x}{3}\right)^n = \sum_{n=0}^{\infty} \left(\frac{x}{3}\right)^n - a_0$$

$$= \frac{1}{1-\frac{x}{3}} - \left(\frac{x}{3}\right)^0$$

$$= \frac{1}{1-\frac{x}{3}} - 1$$

for convergence $\left|\frac{x}{3}\right| < 1$

$|x| < 3$

$-3 < x < 3$