

## Quiz 10

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

**Question 1.** (2 marks) §8.2 #17 Determine whether the series is convergent or divergent. If it is convergent, find its sum.

$$\sum_{n=1}^{\infty} \underbrace{\arctan n}_{a_n} \quad \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \arctan n = \frac{\pi}{2} \neq 0 \quad \therefore \text{diverges by } n^{\text{th}} \text{ term divergence test}$$

**Question 2.** (4 marks) §8.2 #19 Determine whether the series is convergent or divergent by expressing  $S_n$  as a telescoping sum. If it is convergent find its sum.

$$\sum_{n=2}^{\infty} \frac{2}{n^2-1} = \sum_{n=2}^{\infty} \frac{2}{(n-1)(n+1)}$$

$$\frac{2}{(n-1)(n+1)} = \frac{A}{n-1} + \frac{B}{n+1}$$

$$2 = A(n+1) + B(n-1)$$

Let  $n = -1 : B = -1$   
 Let  $n = 1 : A = 1$

$$S_n = a_2 + a_3 + a_4 + a_5 + a_6 + \dots + a_{n-4} + a_{n-3} + a_{n-2} + a_{n-1} + a_n$$

$$= \left[1 - \frac{1}{3}\right] + \left[\frac{1}{2} - \frac{1}{4}\right] + \left[\frac{1}{3} - \frac{1}{5}\right] + \left[\frac{1}{4} - \frac{1}{6}\right] + \left[\frac{1}{5} - \frac{1}{7}\right] + \dots +$$

$$\left[\frac{1}{n-5} - \frac{1}{n-3}\right] + \left[\frac{1}{n-4} - \frac{1}{n-2}\right] + \left[\frac{1}{n-3} - \frac{1}{n-1}\right] + \left[\frac{1}{n-2} - \frac{1}{n}\right] + \left[\frac{1}{n-1} - \frac{1}{n+1}\right]$$

$$= 1 + \frac{1}{2} - \frac{1}{n} - \frac{1}{n+1}$$

$$S = \lim_{n \rightarrow \infty} [S_n] = \lim_{n \rightarrow \infty} \left[1 + \frac{1}{2} - \frac{1}{n} - \frac{1}{n+1}\right] = \frac{3}{2}$$

**Question 3.** (4 marks) §8.1 #28 Find the values of  $x$  for which the series converges. Find the sum of the series for those values of  $x$ .

$$\sum_{n=0}^{\infty} 2^n(x+1)^n = \sum_{n=0}^{\infty} (2(x+1))^n \quad \text{for convergence } |2(x+1)| < 1$$

$$= \frac{1}{1-2(x+1)}$$

$$|x+1| < \frac{1}{2}$$

$$-\frac{1}{2} < x+1 < \frac{1}{2}$$

$$-\frac{3}{2} < x < -\frac{1}{2}$$