

Quiz 11

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. (5 marks) §8.3 #24 Determine whether the series is convergent or divergent.

$$\sum_{n=1}^{\infty} \frac{1 + \sin n}{10^n} \quad \text{Let } a_n = \frac{1 + \sin n}{10^n}$$

$$0 \leq a_n \leq \frac{1+1}{10^n} = 2\left(\frac{1}{10}\right)^n = b_n$$

$\therefore \sum_{n=1}^{\infty} a_n$ is convergent by comparison test since $\sum_{n=1}^{\infty} b_n$

is convergent (geometric series where $|r| = \frac{1}{10} < 1 \therefore$ convergent)

Question 2. (5 marks) §8.4 #37 Determine whether the series is absolutely convergent, conditionally convergent, or divergent.

$$\sum_{n=1}^{\infty} \frac{2 \cdot 4 \cdot 6 \cdot \dots \cdot (2n)}{n!} \quad \text{Let's apply the ratio test. Let } a_n = \frac{2 \cdot 4 \cdot 6 \cdot \dots \cdot (2n)}{n!}$$

$$\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right|$$

$$= \lim_{n \rightarrow \infty} \left| \frac{2 \cdot 4 \cdot 6 \cdot \dots \cdot (2n) \cdot (2(n+1))}{(n+1)!} \cdot \frac{n!}{2 \cdot 4 \cdot 6 \cdot \dots \cdot (2n)} \right|$$

$$= \lim_{n \rightarrow \infty} \frac{2 \cdot 4 \cdot 6 \cdot \dots \cdot (2n) \cdot (2n+2)}{n! \cdot (n+1)} \cdot \frac{n!}{2 \cdot 4 \cdot 6 \cdot \dots \cdot (2n)}$$

$$= \lim_{n \rightarrow \infty} \frac{2n+2}{n+1}$$

$$= 2 > 1 \quad \therefore \text{diverges by ratio test.}$$