Name: *Y. Lamontagne*

Student ID: _____

Quiz 11

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(5 marks)* §8.3 #23 Determine whether the series is convergent or divergent.

$$\sum_{n=1}^{\infty} \frac{2+(-1)^n}{n\sqrt{n}}$$

Let $a_n = \dfrac{2+(-1)^n}{n\sqrt{n}}$

$$0 \le a_n \le \frac{2+1}{n\sqrt{n}} = \frac{3}{n^{3/2}} = b_n$$

$$\therefore \sum_{n=1}^{\infty} a_n \text{ is convergent by comparison test since } \sum_{n=1}^{\infty} b_n$$

is convergent (p-series where $p = \frac{3}{2} > 1$ $\therefore$ convergent).

Question 2. *(5 marks)* §8.4 #38 Determine whether the series is absolutely convergent, conditionally convergent, or divergent.

$$\sum_{n=1}^{\infty} (-1)^n \frac{2^n n!}{5 \cdot 8 \cdot 11 \cdots (3n+2)}$$

Lets apply the ratio test. Let $a_n = \dfrac{(-1)^n 2^n n!}{5(8)(11) \cdots (3n+2)}$

$$\lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right|$$

$$= \lim_{n \to \infty} \left| \frac{\dfrac{(-1)^{n+1} 2^{n+1}(n+1)!}{5(8)(11) \cdots (3n+2)(3(n+1)+2)}}{\dfrac{(-1)^n 2^n n!}{5(8)(11) \cdots (3n+2)}} \right|$$

$$= \lim_{n \to \infty} \frac{2^n 2(n+1)n!}{5(8)(11) \cdots (3n+2)(3n+5)} \cdot \frac{5(8)(11) \cdots (3n+2)}{2^n n!}$$

$$= \lim_{n \to \infty} \frac{2(n+1)}{3n+5} = \frac{2}{3} < 1 \qquad \therefore$$

absolutely converges by ratio test