Dawson College: **Calculus II** (SCIENCE): 201-NYB-05-S2: Winter 2012

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Quiz 12

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(5 marks)* §8.7 #8 Find the Maclaurin series for $f(x) = xe^x$. Assume that f has a power series expansion. Do not show that $R_n \to 0$.

$f(x) = xe^x \qquad f(0) = 0$

$f'(x) = e^x + xe^x \qquad f'(0) = 1$

$f''(x) = e^x + e^x + xe^x = 2e^x + xe^x \qquad f''(0) = 2$

$f'''(x) = 2e^x + e^x + xe^x = 3e^x + xe^x \qquad f'''(0) = 3$

$\vdots \qquad \vdots$

$f^{(n)}(x) = ne^x + xe^x \qquad f^{(n)}(0) = n$

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)x^n}{n!} = \sum_{n=0}^{\infty} \frac{nx^n}{n!} = \sum_{n=1}^{\infty} \frac{nx^n}{n(n-1)!}$$

$$= \sum_{n=1}^{\infty} \frac{x^n}{(n-1)!}$$

Question 2. *(5 marks)* §8.7 #15 Find the Taylor series for $f(x) = \cos x$ centered at $x = \pi$. Assume that f has a power series expansion. Do not show that $R_n \to 0$.

$f(x) = \cos x \qquad f(\pi) = \cos\pi = -1$

$f'(x) = -\sin x \qquad f'(\pi) = -\sin\pi = 0$

$f''(x) = -\cos x \qquad f''(\pi) = -\cos\pi = 1$

$f'''(x) = \sin x \qquad f'''(\pi) = \sin\pi = 0$

$f^{(4)}(x) = \cos x \qquad f^{(4)}(\pi) = \cos\pi = -1$

$f^{(5)}(x) = -\sin x \qquad f^{(5)}(\pi) = -\sin\pi = 0$

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(\pi)(x-\pi)^n}{n!} = f(\pi) + \frac{f'(\pi)(x-\pi)}{1!} + \frac{f''(\pi)(x-\pi)^2}{2!} + \frac{f'''(\pi)(x-\pi)^3}{3!}$$

$$+ \frac{f''''(\pi)(x-\pi)^4}{4!} + \cdots = -1 + \frac{1}{2!}(x-\pi)^2 - \frac{(x-\pi)^4}{4!}$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^{n+1}(x-\pi)^{2n}}{(2n)!}$$