Dawson College: **Calculus II** (SCIENCE): 201-NYB-05-S2: Winter 2012

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Quiz 12

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(5 marks)* §8.7 #7 Find the Maclaurin series for $f(x) = e^{5x}$. Assume that f has a power series expansion. Do not show that $R_n \to 0$.

$f(x) = e^{5x}$ $\quad f(0) = 1$

$f'(x) = 5e^{5x}$ $\quad f'(0) = 5$

$f''(x) = 5^2 e^{5x}$ $\quad f''(0) = 5^2$

$f'''(x) = 5^3 e^{5x}$ $\quad f'''(0) = 5^3$

$\vdots$ $\quad \vdots$

$f^{(n)}(x) = 5^n e^{5x}$ $\quad f^{(n)}(0) = 5^n$

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)\, x^n}{n!} = \sum_{n=0}^{\infty} \frac{5^n x^n}{n!}$$

Question 2. *(5 marks)* §8.7 #15 Find the Taylor series for $f(x) = \sin x$ centered at $x = \pi/2$. Assume that f has a power series expansion. Do not show that $R_n \to 0$.

$f(x) = \sin x$ $\quad f(\pi/2) = \sin \pi/2 = 1$

$f'(x) = \cos x$ $\quad f'(\pi/2) = \cos \pi/2 = 0$

$f''(x) = -\sin x$ $\quad f''(\pi/2) = -\sin \pi/2 = -1$

$f'''(x) = -\cos x$ $\quad f'''(\pi/2) = 0$

$f^{(4)}(x) = \sin x$ $\quad f^{(4)}(\pi/2) = 1$

$f^{(5)}(x) = \cos x$ $\quad f^{(5)}(\pi/2) = 0$

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(\pi/2)(x-\frac{\pi}{2})^n}{n!} = f(\pi/2) + \frac{f'(\frac{\pi}{2})}{1!}(x-\tfrac{\pi}{2}) + \frac{f''(\pi/2)(x-\pi/2)^2}{2!} + \frac{f'''(\pi/2)(x-\frac{\pi}{2})^3}{3!}$$

$$+ \frac{f^{(4)}(\frac{\pi}{2})(x-\frac{\pi}{2})^4}{4!} + \cdots$$

$$= 1 - \frac{(x-\pi/2)^2}{2!} + \frac{(x-\pi/2)^4}{4!} - \frac{(x-\pi/2)^6}{6!} + \cdots$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n (x-\pi/2)^{2n}}{(2n)!}$$