

Quiz 4

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. (3 marks) §5.4 #9 Find the derivative of the function.

$$h(x) = \int_2^{1/x} \arctan t \, dt = f(g(x)) \quad \text{where } f(x) = \int_2^x \arctan t \, dt$$

$$g(x) = \frac{1}{x}$$

$$f'(x) = \arctan x \quad \text{by 2nd FTC}$$

$$g'(x) = \frac{-1}{x^2}$$

$$\begin{aligned} \text{So } h'(x) &= f'(g(x))g'(x) \\ &= \frac{\arctan\left(\frac{1}{x}\right)}{x^2} \end{aligned}$$

Question 2. (2 marks) §5.4 #18 Find the average value of the function on the given interval.

$$f(\theta) = \sec \theta \tan \theta \quad [0, \pi/4]$$

$$\begin{aligned} \frac{1}{b-a} \int_a^b f(\theta) \, d\theta &= \frac{1}{\pi/4 - 0} \int_0^{\pi/4} \sec \theta \tan \theta \, d\theta = \frac{4}{\pi} \left[\sec \theta \right]_0^{\pi/4} \\ &= \frac{4}{\pi} \left[\sec \frac{\pi}{4} - \sec 0 \right] \\ &= \frac{4}{\pi} \left[\sqrt{2} - 1 \right] \end{aligned}$$

Question 3. (2 marks) §5.5 #47 Evaluate the definite integral.

$$\int_{-\pi/6}^{\pi/6} \tan^3 \theta \, d\theta = 0$$

$\sec \tan^3 \theta$ is an odd function. Let $f(\theta) = \tan^3 \theta$

$$f(-\theta) = (\tan(-\theta))^3 = (-\tan \theta)^3 = -(\tan \theta)^3 = -f(\theta)$$

Question 4. (3 marks) §5.5 #62 If f is continuous and $\int_0^9 f(x) \, dx = 4$, find $\int_0^3 x f(x^2) \, dx = \int_0^9 f(u) \frac{du}{2}$

$$\begin{aligned} u &= x^2 \\ du &= 2x \, dx \\ \frac{du}{2} &= x \, dx \end{aligned} = \frac{1}{2} \int_0^9 f(u) \, du$$

$$\begin{aligned} u(0) &= 0 \\ u(3) &= 9 \end{aligned} = \frac{1}{2} \cdot 4$$

$$= 2$$