Name: Y. Lamontagne
Student ID: _____

Quiz 4

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(3 marks)* §5.4 #9 Find the derivative of the function.

$y = \int_3^{\sqrt{x}} \frac{\cos t}{t}\,dt = f(g(x))$ where $f(x) = \int_3^x \frac{\cos t}{t}\,dt$ and $g(x) = \sqrt{x}$

$f'(x) = \frac{\cos x}{x}$ by 2nd FTC

$g'(x) = \frac{1}{2\sqrt{x}}$

So $y' = f'(g(x))g'(x)$

$= \frac{\cos\sqrt{x}}{\sqrt{x}}\left(\frac{1}{2\sqrt{x}}\right)$

$= \frac{\cos\sqrt{x}}{2x}$

Question 2. *(2 marks)* §5.4 #18 Find the average value of the function on the given interval.

$g(x) = \cos x \quad [0, \pi/2]$

$\frac{1}{b-a}\int_a^b g(x)\,dx = \frac{1}{\frac{\pi}{2}-0}\int_0^{\pi/2} \cos x\,dx = \frac{2}{\pi}\left[\sin x\right]_0^{\frac{\pi}{2}}$

$= \frac{2}{\pi}\left[\sin\frac{\pi}{2} - \sin 0\right]$

$= \frac{2}{\pi}$

Question 3. *(2 marks)* §5.5 #47 Evaluate the definite integral.

$\int_{-\pi/2}^{\pi/2} \frac{x^2\sin x}{1+x^6}\,dx = 0$

since $\frac{x^2\sin x}{1+x^6}$ is an odd function. Let $f(x) = \frac{x^2\sin x}{1+x^6}$

$f(-x) = \frac{(-x)^2\sin(-x)}{1+(-x)^6} = \frac{x^2(-\sin x)}{1+x^6} = -\frac{x^2\sin x}{1+x^6} = -f(x).$

Question 4. *(3 marks)* §5.5 #61 If f is continuous and $\int_0^4 f(x)\,dx = 10$, find $\int_0^2 f(2x)\,dx$

$= \int_0^4 f(u)\frac{du}{2}$

$u = 2x$

$du = 2\,dx$

$\frac{du}{2} = dx$

$= \frac{1}{2}\int_0^4 f(u)\,du$

$= \frac{1}{2}\cdot 10 = 5.$

$u(0) = 2(0) = 0$

$u(2) = 2(2) = 4$