Dawson College: Calculus II (SCIENCE): 201-NYB-05-S2: Winter 2012

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Quiz 5

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(4 marks)* §6.1 #14 Evaluate the integral.

$I = \int e^{-\theta}\cos(2\theta)\,d\theta = uv - \int v\,du$

$u = e^{-\theta}$ $\quad du = -e^{-\theta}\,d\theta$

$v = \frac{\sin(2\theta)}{2}$ $\quad dv = \cos(2\theta)\,d\theta$

$$I = \frac{e^{-\theta}\sin(2\theta)}{2} - \int \frac{-e^{-\theta}\sin(2\theta)}{2}\,d\theta$$

$u = e^{-\theta}$ $\quad du = -e^{-\theta}\,d\theta$

$v = \frac{-\cos(2\theta)}{2}$ $\quad dv = \sin(2\theta)\,d\theta$

$$I = \frac{e^{-\theta}\sin(2\theta)}{2} + \frac{1}{2}\left[uv - \int v\,du\right]$$

$$I = \frac{e^{-\theta}\sin(2\theta)}{2} + \frac{1}{2}\left[\frac{-e^{-\theta}\cos(2\theta)}{2} - \int -e^{-\theta}\left(\frac{-\cos(2\theta)}{2}\right)d\theta\right]$$

$$I = \frac{e^{-\theta}\sin(2\theta)}{2} - \frac{e^{-\theta}\cos(2\theta)}{4} - \frac{1}{4}I + C$$

$$I = \frac{4}{5}\left[\frac{e^{-\theta}\sin(2\theta)}{2} - \frac{e^{-\theta}\cos(2\theta)}{4}\right] + C$$

Question 2. *(3 marks)* §6.1 #25 Evaluate the integral.

$\int_1^2 (\ln x)^2\,dx = \left[uv\right]_1^2 - \int_1^2 v\,du$

$u = (\ln x)^2$ $\quad du = \frac{2\ln x}{x}\,dx$

$v = x$ $\quad dv = dx$

$$= \left[x(\ln x)^2\right]_1^2 - \int_1^2 \frac{2x\ln x}{x}\,dx$$

$$= 2(\ln 2)^2 - 1(\ln 1)^2 - 2\int_1^2 \ln x\,dx$$

$u = \ln x$ $\quad du = \frac{1}{x}\,dx$

$v = x$ $\quad dv = dx$

$$= 2(\ln 2)^2 - 2\left[\left[uv\right]_1^2 - \int_1^2 v\,du\right]$$

$$= 2(\ln 2)^2 - 2\left[\left[x\ln x\right]_1^2 - \int_1^2 x\cdot\frac{1}{x}\,dx\right]$$

$$= 2(\ln 2)^2 - 2\left[\left[2\ln 2\right] - \left[1\ln 1\right] - \int_1^2 dx\right]$$

$$= 2(\ln 2)^2 - 4\ln 2 + 2\left[x\right]_1^2 = 2(\ln 2)^2 - 4\ln 2 + 2$$

Question 3. *(3 marks)* §6.2 #5 Evaluate the integral.

$\int_0^{\pi/2} \cos^2\theta\,d\theta = \int_0^{\pi/2} \frac{1+\cos 2\theta}{2}\,d\theta$

$$= \frac{1}{2}\int_0^{\pi/2} 1+\cos 2\theta\,d\theta$$

$$= \frac{1}{2}\left[\theta + \frac{\sin 2\theta}{2}\right]_0^{\pi/2} = \frac{1}{2}\left[\left[\frac{\pi}{2} + \frac{\sin 2\left(\frac{\pi}{2}\right)}{2}\right] - \left[0 + \frac{\sin 2(0)}{2}\right]\right]$$

$$= \frac{\pi}{4}$$