

Quiz 6

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. (5 marks) §6.2 #61 Evaluate the integral by first completing the square.

$$\int \frac{1}{(x^2+2x+2)^2} dx = \int \frac{1}{((x+1)^2+1)^2} dx$$

$$(x^2+2x+1)-1+2 = (x+1)^2+1$$

$$= \int \frac{1}{(\tan^2\theta+1)^2} \sec^2\theta d\theta$$

$$= \int \frac{1}{(\sec^2\theta)^2} \sec^2\theta d\theta$$

$$= \int \cos^2\theta d\theta$$

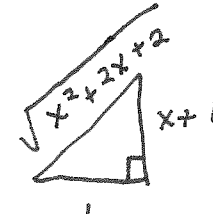
$$= \frac{1}{2} \int 1 + \cos 2\theta d\theta$$

$$= \frac{1}{2} \left[\theta + \frac{\sin 2\theta}{2} \right] + C$$

$$= \frac{1}{2} \left[\theta + \frac{2 \sin\theta \cos\theta}{2} \right] + C$$

$$= \frac{1}{2} \arctan(x+1) + \frac{1}{2} \left(\frac{x+1}{x^2+2x+2} \right) + C$$

$x+1 = \tan\theta$
 $dx = \sec^2\theta d\theta$
 $\theta = \arctan(x+1)$



$\sin\theta = \frac{x+1}{\sqrt{x^2+2x+2}}$
 $\cos\theta = \frac{1}{\sqrt{x^2+2x+2}}$

Question 2. (5 marks) #6 #30 Evaluate the integral.

$$\begin{aligned} \int \frac{x^3}{x^3+1} dx &= \int \frac{x^3+1-1}{x^3+1} dx \\ &= \int \frac{\cancel{x^3+1}}{\cancel{x^3+1}} - \frac{1}{x^3+1} dx \\ &= \int 1 dx - \int \frac{1}{x^3+1} dx \end{aligned}$$

$$\frac{1}{x^3+1} = \frac{1}{(x+1)(x^2-x+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2-x+1}$$

$$1 = A(x^2-x+1) + (Bx+C)(x+1)$$

$$\text{Let } x = -1: 1 = A((-1)^2 - (-1) + 1) + (B(-1) + C)(-1 + 1)$$

$$\frac{1}{3} = A$$

$$\text{Let } x = 0: 1 = A(0^2 - 0 + 1) + (B(0) + C)(0 + 1)$$

$$1 = \frac{1}{3} + C$$

$$\frac{2}{3} = C$$

$$\text{Let } x = 1: 1 = A(1^2 - 1 + 1) + (B(1) + C)(1 + 1)$$

$$1 = \frac{1}{3} + 2B + \frac{4}{3}$$

$$-\frac{1}{3} = -\frac{2}{3} = B$$

$$= x - \int \frac{\frac{1}{3}}{x+1} + \frac{-\frac{1}{3}x + \frac{2}{3}}{x^2-x+1} dx$$

$$= x - \frac{1}{3} \ln|x+1| + \int \frac{\frac{1}{3}x - \frac{2}{3}}{x^2-x+1} dx$$

$$= x - \frac{1}{3} \ln|x+1| + \frac{1}{3} \int \frac{x - \frac{1}{2} - \frac{3}{2}}{x^2-x+1} dx$$

$$= x - \frac{1}{3} \ln|x+1| + \frac{1}{3} \int \frac{x - \frac{1}{2}}{x^2-x+1} dx - \frac{1}{2} \int \frac{1}{x^2-x+1} dx$$

$$= x - \frac{1}{3} \ln|x+1| + \frac{1}{6} \ln(x^2-x+1) - \frac{1}{2} \int \frac{1}{(x-\frac{1}{2})^2 + \frac{3}{4}} dx$$

$$= x - \frac{1}{3} \ln|x+1| + \frac{1}{6} \ln(x^2-x+1) - \frac{1}{2} \sqrt{\frac{4}{3}} \arctan \frac{(x-\frac{1}{2})}{\sqrt{\frac{3}{4}}} + C$$