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Quiz 6

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(5 marks)* §6.2 #62 Evaluate the integral by first completing the square.

$$\int \frac{x^2}{\sqrt{4x-x^2}}\,dx = \int \frac{x^2}{\sqrt{4-(x-2)^2}}\,dx$$

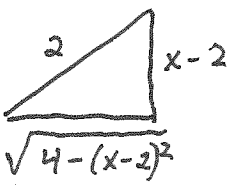
$-x^2+4x$
$= -[x^2-4x]$
$= -[x^2-4x+4-4]$
$= -[(x-2)^2-4]$
$= 4-(x-2)^2$

$x-2 = 2\sin\theta$
$x = 2+2\sin\theta$
$dx = 2\cos\theta\,d\theta$

$\frac{x-2}{2} = \sin\theta$
$\arcsin\left(\frac{x-2}{2}\right) = \theta$

$$= \int \frac{(2+2\sin\theta)^2}{\sqrt{4-(2\sin\theta)^2}}\,2\cos\theta\,d\theta$$

$$= \int \frac{4+8\sin\theta+4\sin^2\theta}{\sqrt{4(1-\sin^2\theta)}}\,2\cos\theta\,d\theta$$



$$= 8\int \frac{1+2\sin\theta+\sin^2\theta}{\sqrt{4\cos^2\theta}}\cos\theta\,d\theta$$

$$= 4\int 1+2\sin\theta+\sin^2\theta\,d\theta$$

$$= 4\int 1+2\sin\theta+\frac{1-\cos 2\theta}{2}\,d\theta$$

$$= 4\left[\theta-2\cos\theta+\frac{1}{2}\theta-\frac{1}{2}\frac{\sin 2\theta}{2}\right]+C$$

$$= 6\theta-8\cos\theta-\sin 2\theta+C$$

$$= 6\theta-8\cos\theta-2\sin\theta\cos\theta+C$$

$$= 6\arcsin\left(\frac{x-2}{2}\right)-\frac{8\sqrt{4-(x-2)^2}}{2}-2\,\frac{x-2}{2}\cdot\frac{\sqrt{4-(x-2)^2}}{2}+C$$

$$= 6\arcsin\left(\frac{x-2}{2}\right)-4\sqrt{4-(x-2)^2}-\frac{(x-2)\sqrt{4-(x-2)^2}}{2}+C$$

Question 2. *(5 marks)* §6.3 #29 Evaluate the integral.

$$\int \frac{1}{x^3-1}\,dx$$

$$\frac{1}{x^3-1} = \frac{1}{(x-1)(x^2+x+1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+x+1}$$

$$1 = A(x^2+x+1) + (Bx+C)(x-1)$$

Let $x=1$: $1 = A(1^2+1+1) + (B(1)+C)(1-1)$

$1/3 = A$

Let $x=0$: $1 = A(0^2+0+1) + (B(0)+C)(0-1)$

$1 = 1/3 - C$

$C = -2/3$

Let $x=-1$: $1 = A((-1)^2+(-1)+1) + (B(-1)+C)(-1-1)$

$1 = A + 2B - 2C$

$1 = 1/3 + 2B - 2(-2/3)$

$1 = 1/3 + 2B + 4/3$

$-1/3 = B$

$$\int \frac{1/3}{x-1} + \frac{(-1/3)x - 2/3}{x^2+x+1}\,dx$$

$$= \frac{1}{3}\ln|x-1| - \frac{1}{3}\int \frac{x+2}{x^2+x+1}\,dx$$

$$= \frac{1}{3}\ln|x-1| - \frac{1}{3}\int \frac{x+\frac{1}{2}+\frac{3}{2}}{x^2+x+1}\,dx$$

$$= \frac{1}{3}\ln|x-1| - \frac{1}{3}\left[\int \frac{x+\frac{1}{2}}{x^2+x+1}\,dx + \frac{3}{2}\int \frac{1}{x^2+x+1}\,dx\right]$$

$$= \frac{1}{3}\ln|x-1| - \frac{1}{6}\ln(x^2+x+1) - \frac{1}{2}\int \frac{1}{(x+\frac{1}{2})^2+\frac{3}{4}}\,dx$$

$$= \frac{1}{3}\ln|x-1| - \frac{1}{6}\ln(x^2+x+1) - \frac{1}{2}\sqrt{\frac{4}{3}}\arctan\left(\frac{x+\frac{1}{2}}{\sqrt{\frac{3}{4}}}\right) + C$$