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Quiz 7

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(5 marks)* §6.6 #48 Find the values of p for which the integral converges and evaluate the integral for those values of p.

$$\int_e^\infty \frac{1}{x(\ln x)^p}\,dx = \lim_{b\to\infty} \int_e^b \frac{1}{x(\ln x)^p}\,dx$$

$u = \ln x$ $\quad = \lim_{b\to\infty} \int_1^{\ln b} \frac{1}{(u)^p}\,du = \lim_{b\to\infty} \int_1^{\ln b} u^{-p}\,du$

$du = \frac{1}{x}dx$

$u(b) = \ln b$

$u(e) = \ln e = 1$

if $p = 1$ then $= \lim_{b\to\infty} \left[\ln|u|\right]_1^{\ln b} = \lim_{b\to\infty} \left[\ln \ln b - \underbrace{\ln 1}_{0}\right]$ ($\ln \ln b \to \infty$)

diverges to ∞

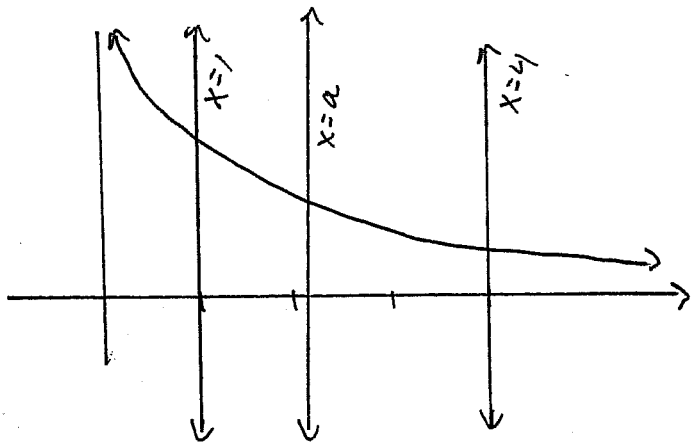
if $p \neq 1$ then $= \lim_{b\to\infty} \left[\frac{u^{-p+1}}{-p+1}\right]_1^{\ln b} = \lim_{b\to\infty} \left[\frac{(\ln b)^{-p+1}}{1-p} - \frac{1}{1-p}\right]$

if $p > 1$ then $\lim_{b\to\infty} \frac{(\ln b)^{-p+1}}{1-p} = 0$ and integral

converges to $\frac{-1}{1-p}$

if $p < 1$ then $\lim_{b\to\infty} \frac{(\ln b)^{-p+1}}{1-p}$ diverges to ∞.

Question 2. *(5 marks)* §7.1 #34a Find the number a such that the line $x = a$ bisects the area under the curve $y = 1/x^2$, $1 \le x \le 4$.



$$\int_1^a \frac{1}{x^2}\,dx = \int_a^4 \frac{1}{x^2}\,dx$$

$$\left[\frac{-1}{x}\right]_1^a = \left[\frac{-1}{x}\right]_a^4$$

$$\frac{-1}{a} + 1 = \frac{-1}{4} + \frac{1}{a}$$

$$\frac{5}{4} = \frac{2}{a}$$

$$a = \frac{8}{5}$$