Name: Y. Lamontagne
Student ID: _____

Quiz 7

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(5 marks)* §6.6 #49 Find the values of p for which the integral converges and evaluate the integral for those values of p.

$$\int_0^1 \frac{1}{x^p}\,dx = \lim_{a\to 0^+} \int_a^1 \frac{1}{x^p}\,dx$$

if $p=1$ then $= \lim_{a\to 0^+} \left[\ln|x|\right]_a^1 = \lim_{a\to 0^+} \left[\ln 1 - \ln|a|\right]$

diverges to ∞

if $p\neq 1$ then $= \lim_{a\to 0^+} \int_a^1 x^{-p}\,dx = \lim_{a\to 0^+} \left[\frac{x^{-p+1}}{1-p}\right]_a^1$

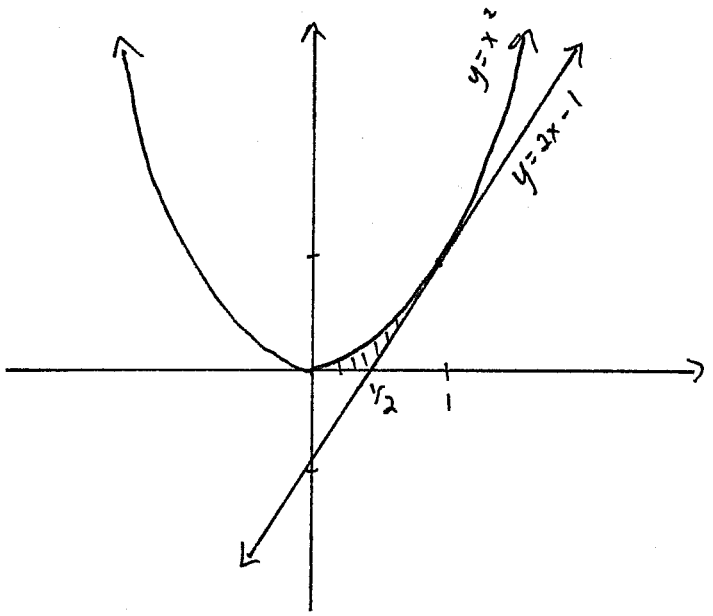
$$= \lim_{a\to 0^+} \left[\frac{1}{1-p} - \frac{a^{-p+1}}{1-p}\right]$$

if $p<1$ then $\lim_{a\to 0^+} \frac{-a^{-p+1}}{1-p} = 0$ and integral

converges to $\frac{1}{1-p}$

if $p>1$ then $\lim_{a\to 0^+} \frac{-a^{-p+1}}{1-p}$ diverges to $-\infty$.

Question 2. *(5 marks)* §7.1 #32 Find the area of the region bounded by the parabola $y = x^2$, the tangent line to this parabola at $(1,1)$, and the x-axis.



$y' = 2x$

slope of tangent at $x = 1$

$m = 2(1)$
$= 2$

So $y = mx + b$
$y = 2x + b$ @ $(1,1)$
$1 = 2(1) + b$
$-1 = b$

$\therefore y = 2x - 1$

$\therefore$ x-int: $0 = 2x - 1$
$\frac{1}{2} = x$

$$\text{Area} = \int_0^{1/2} x^2\,dx + \int_{1/2}^{1} x^2 - (2x-1)\,dx$$

$$= \left[\frac{x^3}{3}\right]_0^{1/2} + \left[\frac{x^3}{3} - \frac{2x^2}{2} + x\right]_{1/2}^{1}$$

$$= \frac{(1/2)^3}{3} + \left[\frac{1}{3} - 1 + 1\right] - \left[\frac{(1/2)^3}{3} - \left(\frac{1}{2}\right)^2 + \frac{1}{2}\right]$$

$$= \frac{1}{3} + \frac{1}{4} - \frac{1}{2}$$

$$= \frac{1}{12}$$