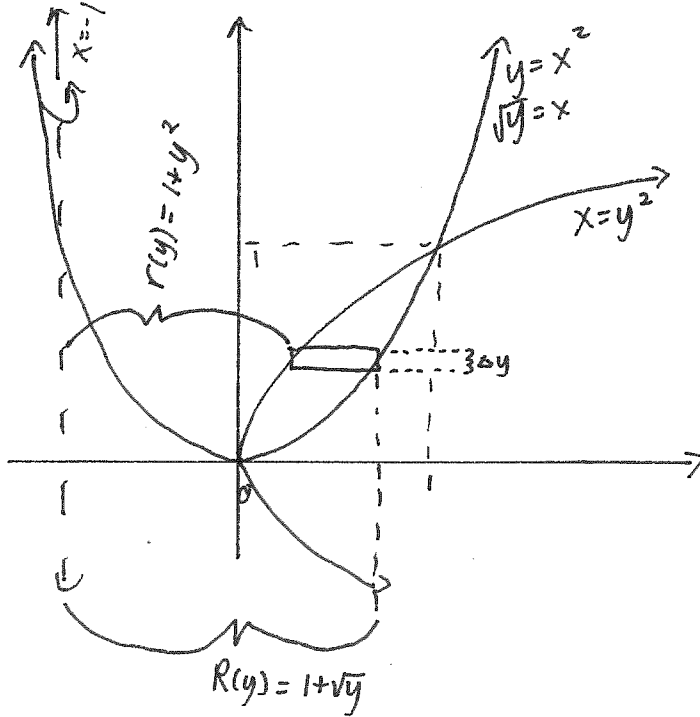


Quiz 8

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. (5 marks) §7.2 #11 Find the volume of the solid obtained by rotating the region bounded by the given curves about the specified line. Sketch the region.

$y = x^2, x = y^2$; about $x = -1$



rep. element:

$$\begin{aligned} \Delta V &= \pi \left[(R(y))^2 - (r(y))^2 \right] \Delta y \\ &= \pi \left[(1 + \sqrt{y})^2 - (1 + y^2)^2 \right] \Delta y \\ &= \pi \left[1 + 2\sqrt{y} + y - 1 - 2y^2 - y^4 \right] \Delta y \\ &= \pi \left[2\sqrt{y} + y - 2y^2 - y^4 \right] \Delta y \end{aligned}$$

$$\begin{aligned} V &= \int_0^1 \pi \left[2\sqrt{y} + y - 2y^2 - y^4 \right] dy \\ &= \pi \left[\frac{4}{3} y^{3/2} + \frac{y^2}{2} - \frac{2y^3}{3} - \frac{y^5}{5} \right]_0^1 \\ &= \pi \left[\frac{4}{3} + \frac{1}{2} - \frac{2}{3} - \frac{1}{5} \right] \\ &= \pi \left[\frac{29}{30} \right] = \frac{29\pi}{30} \end{aligned}$$

Question 2. (5 marks) §7.4 #6 Find the length of the curve.

$y = \frac{x^2}{2} - \frac{\ln x}{4}$ $2 \leq x \leq 4$ $y' = x - \frac{1}{4x}$

$$\begin{aligned} S &= \int_2^4 \sqrt{1 + \left(x - \frac{1}{4x}\right)^2} dx \\ &= \int_2^4 \sqrt{1 + x^2 - \frac{1}{2} + \frac{1}{16x^2}} dx \\ &= \int_2^4 \sqrt{x^2 + \frac{1}{2} + \frac{1}{16x^2}} dx \\ &= \int_2^4 \sqrt{\left(x + \frac{1}{4x}\right)^2} dx \end{aligned}$$

$$\begin{aligned} &= \int_2^4 \left| x + \frac{1}{4x} \right| dx \\ &= \int_2^4 \left(x + \frac{1}{4x} \right) dx \\ &= \left[\frac{x^2}{2} + \frac{1}{4} \ln|x| \right]_2^4 \\ &= \frac{4^2}{2} + \frac{1}{4} \ln 4 - \left[\frac{2^2}{2} + \frac{1}{4} \ln 2 \right] \\ &= 8 + \frac{1}{2} \ln 2 - 2 - \frac{1}{4} \ln 2 = 6 + \frac{1}{4} \ln 2 \end{aligned}$$