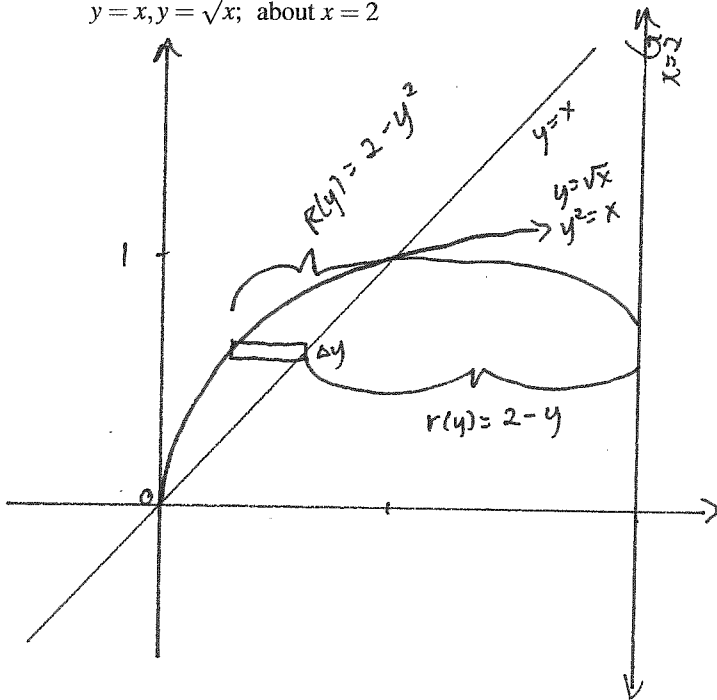


Quiz 8

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. (5 marks) §7.2 #12 Find the volume of the solid obtained by rotating the region bounded by the given curves about the specified line. Sketch the region.

$y = x, y = \sqrt{x};$ about $x = 2$



rep. element:

$$\begin{aligned} \Delta V &= \pi [(R(y))^2 - (r(y))^2] \Delta y \\ &= \pi [(2-y^2)^2 - (2-y)^2] \Delta y \\ &= \pi [4 - 4y^2 + y^4 - 4 + 4y - y^2] \Delta y \\ &= \pi [4y - 5y^2 + y^4] \Delta y \end{aligned}$$

$$\begin{aligned} V &= \int_0^1 \pi [4y - 5y^2 + y^4] dy \\ &= \pi \left[2y^2 - \frac{5y^3}{3} + \frac{y^5}{5} \right]_0^1 \\ &= \pi \left[2 - \frac{5}{3} + \frac{1}{5} \right] \\ &= \frac{8\pi}{15} \end{aligned}$$

Question 2. (5 marks) §7.4 #5 Find the length of the curve.

$y = \frac{x^5}{6} + \frac{1}{10x^3} \quad 1 \leq x \leq 2$

$y' = \frac{5}{6}x^4 - \frac{3}{10x^4}$

$$\begin{aligned} S &= \int_a^b \sqrt{1 + (y')^2} dx \\ &= \int_1^2 \sqrt{1 + \left(\frac{5}{6}x^4 - \frac{3}{10x^4}\right)^2} dx \\ &= \int_1^2 \sqrt{1 + \frac{25}{36}x^8 - \frac{1}{2} + \frac{9}{100x^8}} dx \\ &= \int_1^2 \sqrt{\frac{25}{36}x^8 + \frac{1}{2} + \frac{9}{100x^8}} dx \\ &= \int_1^2 \sqrt{\left(\frac{5}{6}x^4 + \frac{3}{10x^4}\right)^2} dx \end{aligned}$$

$$\begin{aligned} &= \int_1^2 \left| \frac{5}{6}x^4 + \frac{3}{10x^4} \right| dx \\ &= \int_1^2 \left(\frac{5}{6}x^4 + \frac{3}{10x^4} \right) dx \\ &= \left[\frac{1}{6}x^5 - \frac{1}{10x^3} \right]_1^2 \\ &= \frac{2^5}{6} - \frac{1}{10 \cdot 2^3} - \frac{1}{6} + \frac{1}{10} \\ &= \frac{16}{3} - \frac{1}{80} - \frac{1}{6} + \frac{1}{10} = \frac{1261}{240} \end{aligned}$$