

Name: _____
Student ID: _____

Test 2

This test is graded out of 45 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. (5 marks) Evaluate the integral:

$$\int e^{\pi x} \tan^3(e^{\pi x}) \sec^5(e^{\pi x}) dx$$

Question 2. (5 marks) Evaluate the integral:

$$\int \frac{x^2 + 2bx + b^2}{(a^2 - b^2 - 2bx - x^2)^{3/2}} dx$$

Question 3. (5 marks) Evaluate the integral:

$$\int \frac{x^3 - 2x^2 + x + 1}{x^4 + 5x^2 + 4} dx$$

Question 4. (5 marks) Evaluate the integral:

$$\int_0^{\frac{\pi}{4}} \frac{\sec^2 \phi}{\sqrt{1 - \tan^2 \phi}} d\phi$$

Question 5. (5 marks) Find the values of $\kappa \neq -1$ for which the integral converges and evaluate the integral for those values of κ .

$$\int_1^{\infty} x^{\kappa} \ln x \, dx$$

Question 6. (5 marks) Sketch the region in the first quadrant enclosed by $f(x) = \operatorname{arcsec} x$, the tangent line to $f(x)$ at $(2, \frac{\pi}{3})$, the x -axis and y -axis. Set up the integral to find the area enclosed but *do not* evaluate.

Question 7. (5 marks) Set up the integral to find the volume of the solid obtained from the region in the first quadrant bounded by the graphs of $y = \cos x$, $y = \frac{4x}{\pi\sqrt{2}}$ and $x = 0$ rotated about the line $x = \frac{\pi}{4}$.

Question 8. (5 marks) Set up the integral to find the volume of the solid obtained from the region bounded by the graphs of $x = y^2 - 2y$, $x = y$ rotated about the line $x = -1$.

Question 9. (5 marks) Find the length of the curve $y = \ln(\sec x)$ on $[0, \frac{\pi}{4}]$.

Bonus Question. (5 marks) Find the volume of a solid whose base is an elliptical region with boundary curve $a^2x^2 + b^2y^2 = a^2b^2$. Cross-sections perpendicular to the x -axis are isosceles right triangles with hypotenuse in the base.