Dawson College: **Calculus II** (SCIENCE): 201-NYB-05-S2: Winter 2012

Name: ____________________
Student ID: ____________________

Test 3

This test is graded out of 45 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1.

a. *(4 marks)* Show that the series

$$\sum_{n=1}^{\infty} \frac{n^n}{(2n)!}$$

is convergent

b. *(1 mark)* Deduce that

$$\lim_{n\to\infty} \frac{n^n}{(2n)!} = 0.$$

Question 2. *(5 marks)* Find the radius of convergence and interval of convergence of the series

$$\sum_{n=1921}^{\infty} \frac{2^n (x-7)^n}{\sqrt{n-17}}$$

Question 3. *(5 marks)* Determine whether the series is convergent or divergent. If it is convergent find its sum.

$$\sum_{n=3}^{\infty} \ln\left(\frac{\sec\left(\frac{\pi}{n}\right)}{\sec\left(\frac{\pi}{n+1}\right)}\right)$$

Question 4. *(5 marks)* Determine whether the series is absolutely convergent, conditionally convergent, or divergent.

$$\sum_{n=4}^{\infty} \frac{(-1)^n \tan\left(\frac{\pi}{n}\right)}{1+(1.1)^n}$$

Question 5. *(5 marks)* Determine whether the series is convergent or divergent.

$$\sum_{n=1}^{\infty} \frac{n! \tan\left(\frac{1}{n}\right)}{(n-1)!}$$

Question 6.

a. *(1 mark)* Find a formula for the general term a_n of the sequence, assuming that the pattern of the first few terms continues.

$$\left\{\frac{9}{\pi}, \frac{16}{\pi^2}, \frac{25}{\pi^3}, \frac{36}{\pi^4}, \frac{49}{\pi^5}, \dots\right\}_{n=1}^{\infty}$$

b. *(1 mark)* Show that a_n is monotonic.

c. *(1 mark)* Show that a_n is bounded.

d. *(1 mark)* By which theorem can we conclude that a_n converges.

e. *(1 mark)* Determine the limit of a_n as $n \to \infty$.

Question 7. *(5 marks)* Find the values of $\kappa > 0$ for which the series is convergent.

$$\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^{\kappa}}$$

Question 8. *(5 marks)* What is the value of ζ if

$$\sum_{n=1}^{\infty} \sqrt{2}(\ln \zeta)^n = \pi$$

Question 9. *(5 marks)* Determine whether the series is absolutely convergent, conditionally convergent, or divergent.

$$\sum_{n=1871}^{\infty} \frac{(-1)^n(n+1)}{\sqrt{n^4-2n^2+1}}$$

Bonus Question.

a. *(1 mark)* State the $K(\varepsilon)$ definition of the limit of a sequence.

b. *(4 marks)* Use the $K(\varepsilon)$ definition of the limit to prove the squeeze theorem.