

Assignment: Natural Deduction (Contradiction)*

Question 1. (19 marks) Provide a justification (rule and line numbers) for each line of these proofs.

1	$\neg C \Rightarrow B \vee C$	Premise
2	$A \wedge \neg B$	hyp for $\rightarrow I$
3	$\neg C$	hyp for $\neg I$
4	$B \vee C$	1, 3, $\rightarrow E$
5	B	3, 4, $\vee E$
6	$\neg B$	2, $\wedge E$
7	$B \wedge \neg B$	5, 6, $\wedge I$ \downarrow
8	$\neg\neg C$	3-7, $\neg I$
9	C	8, DN
10	$(A \wedge \neg B) \Rightarrow C$	2-9, $\rightarrow I$

1	$(P \vee \neg Q) \Rightarrow \neg R$	Premise
2	$Q \Rightarrow P$	Premise
3	R	hyp for $\neg I$
4	Q	hyp for $\neg I$
5	P	4, 2, $\rightarrow E$
6	$P \vee \neg Q$	5, $\vee I$
7	$\neg R$	6, 1, $\rightarrow E$
8	$R \wedge \neg R$	3, 7, $\wedge I$ \downarrow
9	$\neg Q$	4-8, $\neg I$
10	$P \vee \neg Q$	9, $\vee I$
11	$\neg R$	10, 1, $\rightarrow E$
12	$R \wedge \neg R$	3, 11, $\wedge I$ \downarrow
13	$\neg R$	3-12, $\neg I$

*from Proofs and Concepts: the fundamentals of abstract mathematics by Dave Witte Morris and Joy Morris

$(B \rightarrow C) \rightarrow D, (D \wedge E) \rightarrow F, \neg((E \vee F) \wedge A), A$

0 0 $\neg(A \leftrightarrow (B \rightarrow C))$

1	$(B \rightarrow C) \rightarrow D$	Premise
2	$(D \wedge E) \rightarrow F$	Premise
3	$\neg((E \vee F) \wedge A)$	Premise
4	A	Premise
5	$A \leftrightarrow (B \rightarrow C)$	hyp. for $\neg I$
6	$A \rightarrow (B \rightarrow C)$	5, $\leftrightarrow E$
7	$B \rightarrow C$	4, 6, $\rightarrow E$
8	$\neg(E \vee F) \vee \neg A$	3, DM
9	$\neg(E \vee F)$	4, 8, $\vee E$
10	$\neg E \wedge \neg F$	9, DM
11	$\neg E$	10, $\wedge E$
12	$\neg F$	10, $\wedge E$
13	$\neg(D \wedge E)$	12, 2, MT
14	$\neg D \vee \neg E$	13, DM
15	$\neg D$	11, 14, $\vee E$
16	$\neg(B \rightarrow C)$	15, 1, MT
17	$(B \rightarrow C) \wedge \neg(B \rightarrow C)$	7, 16, $\wedge I$ \downarrow
18	$\neg(A \leftrightarrow (B \rightarrow C))$	5-17, $\neg I$