

Assignment: Natural Deduction (Sub Proof)*

Question 1. (20 marks) Provide a justification (rule and line numbers) for each line of these proofs.

1	$A \Rightarrow B$	Premise
2	$\neg A \Rightarrow C$	Premise
3	$A \vee \neg A$	using result shown in class
4	A	hypothesis for $\rightarrow I$
5	B	4, 1, $\rightarrow E$
6	$B \vee C$	5, $\vee I$
7	$A \Rightarrow (B \vee C)$	4-6, $\rightarrow I$
8	$\neg A$	hypothesis for $\rightarrow I$
9	C	8, 2, $\rightarrow E$
10	$B \vee C$	9, $\vee I$
11	$\neg A \Rightarrow (B \vee C)$	8-10, $\rightarrow I$
12	$B \vee C$	3, 7, 11, $\vee E$

1	$Z \Rightarrow (C \wedge \neg N)$	Premise
2	$\neg Z \Rightarrow (N \wedge \neg C)$	Premise
3	$Z \vee \neg Z$	using result shown in class
4	Z	hypothesis for $\rightarrow I$
5	$C \wedge \neg N$	4, 1, $\rightarrow E$
6	C	5, $\wedge E$
7	$N \vee C$	6, $\vee I$
8	$Z \Rightarrow (N \vee C)$	4-7, $\rightarrow I$
9	$\neg Z$	hypothesis for $\rightarrow I$
10	$N \wedge \neg C$	9, 2, $\rightarrow E$
11	N	10, $\wedge E$
12	$N \vee C$	11, $\vee I$
13	$\neg Z \Rightarrow (N \vee C)$	9-12, $\rightarrow I$
14	$N \vee C$	3, 8, 13, $\vee E$

1	$P \rightarrow (Q \rightarrow R)$	Premise
2	$\neg Q \rightarrow \neg S$	Premise
3	$S \vee T$	Premise
4	$\neg T \vee U$	Premise
5	$\neg U$	Premise

6	P hypothesis for $\rightarrow I$	
7	$Q \rightarrow R$	1, 6, $\rightarrow E$
8	$\neg T$	4, 5, $\vee E$
9	S	3, 8, $\vee E$
10	$\neg \neg S$	9, DN
11	$\neg \neg Q$	2, 10, MT
12	Q	10, DN
13	R	12, 7, $\rightarrow E$
14	$P \rightarrow R$	6-13, $\rightarrow I$
15	$(P \rightarrow R) \vee Q$	14, $\vee I$