

Quiz: Natural Deduction

No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. Only using the rules of inference and the rules of replacement provided show the following are valid arguments using Fitch style natural deduction:

- a. (10 marks) $(\neg P \wedge \neg Q) \rightarrow (Q \vee S), Q \rightarrow P, \neg S \vee L, \therefore \neg P \rightarrow L$
- b. (10 marks) $(G \vee H) \rightarrow \neg P, \neg W \wedge (G \vee Y), Y \rightarrow W, \therefore \neg P$
- c. (10 marks) $\neg P \rightarrow (V \vee M), \neg S \rightarrow (L \wedge \neg M), (V \wedge L) \leftrightarrow S, \therefore P \vee S$

a)

1	$(\neg P \wedge \neg Q) \rightarrow (Q \vee S)$	Premise
2	$Q \rightarrow P$	Premise
3	$\neg S \vee L$	Premise
4	$\neg P$	hypothesis for $\rightarrow I$
5	$\neg Q$	4, 2, MT
6	$\neg P \wedge \neg Q$	4, 5, $\wedge I$
7	$Q \vee S$	6, 1, $\rightarrow E$
8	S	7, 5, $\vee E$
9	$\neg \neg S$	8, DN
10	L	9, 3, $\vee E$
11	$\neg P \rightarrow L$	4-10, $\rightarrow I$

b)

1	$(G \vee H) \rightarrow \neg P$	Premise
2	$\neg W \wedge (G \vee Y)$	Premise
3	$Y \rightarrow W$	Premise
4	$\neg W$	2, $\wedge E$
5	$G \wedge Y$	2, $\wedge E$
6	$\neg Y$	3, 4, MT
7	G	5, 6, $\vee E$
8	$G \vee H$	7, $\vee I$
9	$\neg P$	1, 8, $\rightarrow E$

c)

1	$\neg P \rightarrow (V \vee M)$	Premise
2	$\neg S \rightarrow (L \wedge \neg M)$	Premise
3	$(V \wedge L) \leftrightarrow S$	Premise
4	$\neg(P \vee S)$	hypothesis for $\neg I$
5	$\neg P \wedge \neg S$	4, DN
6	$\neg P$	5, $\wedge E$
7	$\neg S$	5, $\wedge E$
8	$V \vee M$	6, 1, $\rightarrow E$
9	$L \wedge \neg M$	7, 2, $\rightarrow E$
10	$(V \wedge L) \rightarrow S$	3, $\leftrightarrow E$
11	$\neg(V \wedge L)$	7, 10, MT
12	$\neg V \vee \neg L$	11, DN
13	L	9, $\wedge E$
14	$\neg M$	9, $\wedge E$
15	$\neg \neg L$	13, DN
16	$\neg V$	15, 12, $\vee E$
17	M	16, 8, $\vee E$
18	$M \wedge \neg M$	14, 17, $\wedge I$
19	$\neg(P \vee S)$	4-18, $\neg I$
20	$P \vee S$	19, DN