Dawson College: Principles of Mathematics and Logic: 360-124-DW-S01: Winter 2013

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Test 1

No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(2 marks)* Name a logician and state his main contribution to logic.

- see slides.

Question 2.[1] Given the following symbolization key:
A: Alexander Berkman loves Emma Goldman
B_1: Alexander Berkman buys bread.
B_2: Emma Golman buys bread.
E: Emma Goldman loves Alexander Berkman.
F_1: Alexander Berkman buys flowers.
F_2: Emma Goldman buys flowers.
P_1: Alexander Berkman protests.
P_2: Emma Goldman protests.

Translate each English language statement into Propositional Logic.

a. *(3 marks)* Emma and Alexander love each other only if, it is the case that both Emma and Alexander protest.

b. *(3 marks)* Emma buys flowers and Aleaxander buys bread if, neither Alexander loves Emma nor Emma loves Alexander.

Translate each Propositional Logic statement into English.

c. *(1 mark)* $\neg P_2$

d. *(3 marks)* $(\neg P_2 \wedge B_2) \iff E$

a) $(E \wedge A) \rightarrow (P_1 \wedge P_2)$ b) $(\neg A \wedge \neg E) \rightarrow (F_2 \wedge B_1)$

c) Emma Goldman does not protest

d) Emma Goldman buys bread and does not protest if and only if she loves Alexander Berkman.

[1] not historically accurate

Question 3. *(6 marks)* Determine wether the following statement is a tautology, contradiction, or contingent statement. Justify your conclusion.

$[(\neg A \to B) \wedge (\neg A \to \neg B)] \to A$ = ψ

A	B	$\neg A$	$\neg B$	$\neg A \to B$	$\neg A \to \neg B$	$(\neg A \to B) \wedge (\neg A \to \neg B)$	ψ
T	T	F	F	T	T	T	T
T	F	F	T	T	T	T	T
F	T	T	F	T	F	F	T
F	F	T	T	F	T	F	T

The statement is a tautology since under all valuation the statement is true.

Question 4. *(6 marks)* Determine whether the following is a valid argument. Justify your conclusion.

$\neg F_2, (\neg P_2 \wedge B_2) \iff E \therefore E$

The argument is invalid since we can find a valuation where the premises are true and the conclusion is false.

That is
- E false
- $\neg F_2$ true so F_2 false
- It follows that $(\neg P_2 \wedge B_2) \Leftrightarrow E$ is true if B_2 is false.

$(\neg P_2 \wedge F) \Leftrightarrow F = F \Leftrightarrow F = T$

Question 5. Which of the following is possible? If it is possible, give an example. If it is not possible, explain why.

a. *(3 marks)* A valid argument, the conclusion of which is a tautology.

b. *(3 marks)* An invalid argument, the conclusion of which is a tautology.

a) This is possible: $A \therefore \neg A \vee A$

tautology (always true) Therefore impossible to have true premise and false conclusion.

b) This is not possible since for an argument to be invalid there must exist a valuation which make the premises true and conclusion false. But since the conclusion is a tautology no valuation will make it false.

Bonus Question. *(1 mark)* Why did you choose to study Math ~~in the Liberal Arts program~~?

Because it is cool!