

Rules of Inference

Repeat (or R)	$\Phi \therefore \Phi$
\rightarrow-elimination (or \rightarrowE) (or <i>Modus ponens</i>)	$\Phi \rightarrow \Psi, \Phi \therefore \Psi$
Modus tollens (or MT)	$\Phi \rightarrow \Psi, \neg\Psi \therefore \neg\Phi$
\leftrightarrow-introduction (or \leftrightarrowI) (or <i>Biconditional introduction</i>)	$\Phi \rightarrow \Psi, \Psi \rightarrow \Phi \therefore \Phi \leftrightarrow \Psi$
\leftrightarrow-elimination (or \leftrightarrowE) (or <i>Biconditional elimination</i>)	$\Phi \leftrightarrow \Psi \therefore \Phi \rightarrow \Psi$ and $\Phi \leftrightarrow \Psi \therefore \Psi \rightarrow \Phi$
\wedge-introduction (or \wedgeI) (or <i>Conjunction introduction</i>)	$\Phi, \Psi \therefore \Phi \wedge \Psi$
\wedge-elimination (or \wedgeE) (or <i>Simplification</i>)	$\Phi \wedge \Psi \therefore \Phi$ and $\Phi \wedge \Psi \therefore \Psi$
\vee-introduction (or \veeI) (or <i>Disjunction introduction, Addition</i>)	$\Phi \therefore \Phi \vee \Psi$ and $\Psi \therefore \Phi \vee \Psi$
Disjunction elimination (or DE)	$\Phi \rightarrow \Psi, \Theta \rightarrow \Psi, \Phi \vee \Theta \therefore \Psi$
\vee-elimination (or \veeE) (or <i>Disjunctive syllogism</i>)	$\Phi \vee \Psi, \neg\Phi \therefore \Psi$ and $\Phi \vee \Psi, \neg\Psi \therefore \Phi$
Hypothetical syllogism (or HS)	$\Phi \rightarrow \Psi, \Psi \rightarrow \Theta \therefore \Phi \rightarrow \Theta$
Constructive dilemma (or CD)	$\Phi \rightarrow \Psi, \Theta \rightarrow \Pi, \Phi \vee \Theta \therefore \Psi \vee \Pi$
Destructive dilemma (or DD)	$\Phi \rightarrow \Psi, \Theta \rightarrow \Pi, \neg\Psi \vee \neg\Pi \therefore \neg\Phi \vee \neg\Theta$
Absorption (or ABS)	$\Phi \rightarrow \Psi \therefore \Phi \rightarrow (\Phi \wedge \Psi)$ and $\Phi \rightarrow \Psi \therefore \Phi \rightarrow (\Psi \wedge \Phi)$

Rules of Replacement

Associativity (or Asso.)	$\Phi \square (\Psi \square \Theta) \equiv (\Phi \square \Psi) \square \Theta$ where $\square \in \{\wedge, \vee, \leftrightarrow\}$
Commutativity (or Comm.)	$\Phi \square \Psi \equiv \Psi \square \Phi$ where $\square \in \{\wedge, \vee, \leftrightarrow\}$
Distributivity (or Dist.)	$\Phi \wedge (\Psi \vee \Theta) \equiv (\Phi \wedge \Psi) \vee (\Phi \wedge \Theta)$ and $\Phi \vee (\Psi \wedge \Theta) \equiv (\Phi \vee \Psi) \wedge (\Phi \vee \Theta)$
Double negation (or DN)	$\neg\neg\Phi \equiv \Phi$
De Morgan's laws (or DM)	$\neg(\Phi \vee \Psi) \equiv \neg\Phi \wedge \neg\Psi$ and $\neg(\Phi \wedge \Psi) \equiv \neg\Phi \vee \neg\Psi$
Transposition (or Trans.)	$\Phi \rightarrow \Psi \equiv \neg\Psi \rightarrow \neg\Phi$
Material implication (or MI)	$\Phi \rightarrow \Psi \equiv \neg\Phi \vee \Psi$
Biconditional implication (or BI)	$\Phi \leftrightarrow \Psi \equiv (\Phi \rightarrow \Psi) \wedge (\Psi \rightarrow \Phi)$
Exportation (or Expo.)	$(\Phi \wedge \Psi) \rightarrow \Theta \equiv \Phi \rightarrow (\Psi \rightarrow \Theta)$
Tautology (or Taut.)	$\Phi \square \Phi \equiv \Phi$ where $\square \in \{\wedge, \vee\}$