Dawson College: Principles of Mathematics and Logic: 360-124-DW: Winter 2013

Name: Y. Lamontagne
Student ID:

Test 2

No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. Given the following symbolization key:
A: Alexander Berkman loves Emma Goldman
E: Emma Goldman loves Alexander Berkman.
F_1: Alexander Berkman buys flowers.
F_2: Emma Goldman buys flowers.
P_1: Alexander Berkman protests.
P_2: Emma Goldman protests.

a. *(1 mark)* Translate the English statement into a propositional logic statement:
Emma Goldman does not love Alexander Berkman if Alexander does not buy flowers.

$$\neg F_1 \to \neg E$$

b. *(1 mark)* Rewrite the propositional logic statement of part a. into a logically equivalent statement using the logical connective 'or'.

$$\neg\neg F_1 \vee \neg E \equiv F_1 \vee \neg E$$

c. *(1 mark)* Give the logical negation of the statement of part b. and distribute the negation using De Morgan Laws.

$$\neg(F_1 \vee \neg E) \equiv \neg F_1 \wedge \neg\neg E \equiv \neg F_1 \wedge E$$

d. *(1 mark)* Translate the propositional logic statement of part c. into an English statement.

Emma Goldman loves Alexander Berkman but Alexander does not byy flowers

Question 2. *(6 marks)* Using a truth table: determine whether the following two statements are logically equivalent. Justify.

$\neg A \to B$

and

$(\neg A \to \neg B) \wedge (B \to \neg A)$

A	B	$\neg A$	$\neg B$	$\neg A \to \neg\neg B$	$B \to \neg A$	$(\neg A \to \neg\neg B) \wedge (B \to \neg A)$	$\neg A \to B$
T	T	F	F	T	F	F	T
T	F	F	T	T	T	T	T
F	T	T	F	F	T	F	T
F	F	T	T	T	T	T	F

The two statements are not logically equivalent since the two columns of the statements are not identical.

Question 3. *(1 marks per correct line - no part marks)* Provide a justification (rule and line number(s)) for **each line** of the following proof.

1	$(A \leftrightarrow B) \vee B$	Premise
2	$\neg B$	hyp for $\to$I
3	$\neg(\neg A \vee C)$	hyp for $\neg$I
4	$\neg\neg A \wedge \neg C$	3, DM
5	$A \leftrightarrow B$	1, 2, $\vee$E
6	$A \to B$	5, $\leftrightarrow$E
7	$\neg\neg A$	4, $\wedge$E
8	A	7, DN
9	B	6, 8, $\to$E
10	$B \wedge \neg B$	2, 9, $\wedge$I ↯
11	$\neg\neg(\neg A \vee C)$	3-10, $\neg$I
12	$\neg A \vee C$	11, DN
13	$\neg B \to (\neg A \vee C)$	2-12, $\to$I
14	$\neg\neg B \vee (\neg A \vee C)$	13, MI
15	$(\neg\neg B \vee (\neg A \vee C)) \vee D$	14, $\vee$I

Question 4. *(10 marks)* Using only the rules of inference and the rules of replacement show that the following argument is valid using Fitch style natural deduction:

$P \to Q,\ \neg P \to R,\ (Q \vee R) \to S,\ \therefore S\vee$

1	$P \to Q$	Premise
2	$\neg P \to R$	Premise
3	$(Q \vee R) \to S$	Premise
4	$\neg S$	hyp for $\neg$I
5	$\neg(Q \vee R)$	3, 4, MT
6	$\neg Q \wedge \neg R$	5, DM
7	$\neg Q$	6, $\wedge$E
8	$\neg R$	6, $\wedge$E
9	$\neg\neg P$	2, 8, MT
10	$\neg P$	1, 7, MT
11	$\neg P \wedge \neg\neg P$	9, 10, $\wedge$I ↯
12	$\neg\neg S$	4-11, $\neg$I
13	S	12, DN

Question 5. *(15 marks)* **Choose one of the argument** and show that it is valid using only the rules of inference and the rules of replacement and Fitch style natural deduction:

$(\neg A \vee B) \leftrightarrow C,\ \neg(A \wedge G) \leftrightarrow \neg D,\ \neg(D \wedge E),\ \neg(C \vee F),\ \therefore E \rightarrow \neg G$

or

$(Y \vee \neg Z) \rightarrow \neg X,\ X \wedge A,\ \neg(A \wedge (B \vee \neg C)),\ ((A \wedge \neg B) \vee H) \rightarrow W,\ \therefore W \wedge Z$

1	$(\neg A \vee B) \leftrightarrow C$	Premise
2	$\neg(A \wedge G) \leftrightarrow \neg D$	Premise
3	$\neg(D \wedge E)$	Premise
4	$\neg(C \vee F)$	Premise
5	E hyp for $\rightarrow I$	
6	$\neg D \vee \neg E$	3, DM
7	$\neg C \wedge \neg F$	4, DM
8	$\neg C$	7, $\wedge E$
9	$\neg F$	7, $\wedge E$
10	$\neg D$	5, 6, $\vee E$
11	$\neg D \rightarrow \neg(A \wedge G)$	2, $\leftrightarrow E$
12	$\neg(A \wedge G)$	10, 11, $\rightarrow E$
13	$\neg A \vee \neg G$	12, DM
14	$(\neg A \vee B) \rightarrow C$	1, $\leftrightarrow E$
15	$\neg(\neg A \vee B)$	8, 14, MT
16	$\neg\neg A \wedge B$	15, DM
17	$\neg\neg A$	16, $\wedge E$
18	$\neg G$	17, 13, $\vee E$
19	$E \rightarrow \neg G$	5 - 18, $\rightarrow I$

1	$(Y \vee \neg Z) \rightarrow \neg X$	Premise
2	$X \wedge A$	Premise
3	$\neg(A \wedge (B \vee \neg C))$	Premise
4	$((A \wedge \neg B) \vee H) \rightarrow W$	Premise
5	X	2, $\wedge E$
6	A	2, $\wedge E$
7	$\neg A \vee \neg(B \vee \neg C)$	3, DM
8	$\neg(B \vee \neg C)$	6, 7, $\vee E$
9	$\neg B \wedge \neg\neg C$	8, DM
10	$\neg B$	9, $\wedge E$
11	$A \wedge \neg B$	6, 10, $\wedge I$
12	$(A \wedge \neg B) \vee H$	11, $\vee I$
13	W	4, 12, $\rightarrow E$
14	$\neg(Y \vee \neg Z)$	1, 5,
15	$\neg Y \wedge \neg\neg Z$	14, DM
16	$\neg\neg Z$	15, $\wedge E$
17	Z	16, DN
18	$W \wedge Z$	13, 17, $\wedge I$