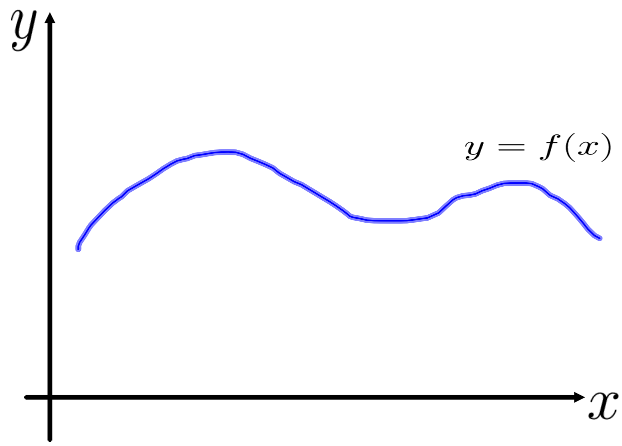
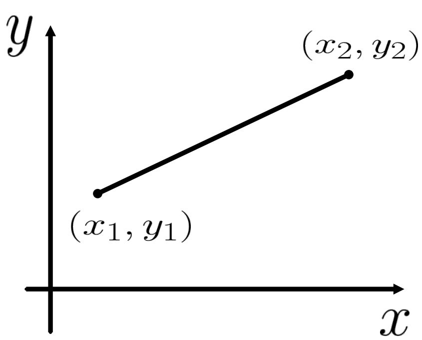
Arc Length

Suppose a curve C is defined by the equation $y = f(x)$ where f is continuous on [a,b].

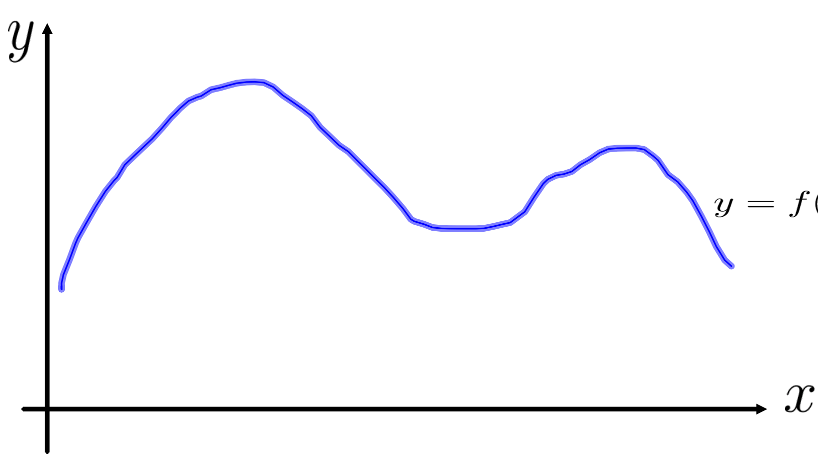


How do we calculate the length of this curve?

Since we know how to calculate the distance between two points:



We can approximate the length of C using line segments:



So, the length of C $\approx \sum_{i=1}^{n}$ (distance between p_i and p_{i-1})

$$\approx \sum_{i=1}^{n} |p_{i-1}\, p_i|$$

As we saw before

We will need the mean value theorem for derivatives:

If f is continuous on [a,b] and differentiable on (a,b) then there is a number c in (a,b) such that

$$f(b) - f(a) = f'(c)(b-a)$$

In our situation, this tells us:

Arc Length Formula

If f' is continuous on [a,b], then the length of $y = f(x),\ a \leq x \leq b$ is

$$L = \int_a^b \sqrt{1 + [f'(x)]^2}\, dx$$

or in Leibniz notation

$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

Examples: 1) Find the length of the curve given by $y^2 = x^3$ between the points (1,1) and (4,8).

2) Find the arc length of the curve $f(x) = x^2 - \frac{1}{8}\ln x$ from $1 \leq x \leq 2$

3) Find the length of the curve $x = y^2$ from (1,-1) to (1,1)

4) Find the length of the curve $y = \ln x$ from x=1 to x=e.