April 16, 2013

Last Name: SOLUTIONS

First Name: _____

Student ID: _____

Quiz 10

Question 1. *(5 marks)* Find the exact length of $x = \frac{y^4}{8} + \frac{1}{4y^2}$ on $1 \leq y \leq 2$ (use the variables as given).

$$\frac{dx}{dy} = \frac{y^3}{2} - \frac{1}{2y^3} \Rightarrow 1 + \left(\frac{dx}{dy}\right)^2 = 1 + \left(\frac{1}{4}y^6 - \frac{1}{2} + \frac{1}{4}y^{-6}\right)$$

$$= \frac{1}{4}y^6 + \frac{1}{2} + \frac{1}{4}y^{-6} = \left(\frac{1}{2}y^3 + \frac{1}{2}y^{-3}\right)^2$$

$$\therefore L = \int_1^2 \sqrt{\left(\frac{1}{2}y^3 + \frac{1}{2}y^{-3}\right)^2}\, dy = \int_1^2 \frac{1}{2}y^3 + \frac{1}{2}y^{-3}\, dy$$

$$= \left[\frac{1}{8}y^4 - \frac{1}{4}y^{-2}\right]_1^2 = \left(2 - \frac{1}{16}\right) - \left(\frac{1}{8} - \frac{1}{4}\right) = 2 + \frac{1}{16}$$

$$= \frac{33}{16}$$

Question 2. *(5 marks)* Find the exact length of $y = \ln(\cos x)$ on $0 \leq x \leq \frac{\pi}{4}$

$$\frac{dy}{dx} = \frac{1}{\cos x} \cdot (-\sin x) = -\tan x \quad \Rightarrow \quad 1 + \left(\frac{dy}{dx}\right)^2 = 1 + \tan^2 x = \sec^2 x$$

$$\therefore L = \int_0^{\pi/4} \sqrt{\sec^2 x}\, dx = \int_0^{\pi/4} \sec x\, dx = \left[\ln|\sec x + \tan x|\right]_0^{\pi/4}$$

SINCE $\sec x \geq 0$ ON $0 \leq x \leq \pi/4$

$$= \ln(1 + \sqrt{2}) - \ln(1 + 0)$$

$$= \ln(1 + \sqrt{2})$$