Dawson College: Calculus II: 201-NYB-05

April 23, 2013

Last Name: SOLUTIONS

First Name: ____________

Student ID: ____________

Quiz 11

Question 1. *(5 marks)* Determine whether the sequence is convergent or not.

$$a_n = \frac{(-10)^{n+1}}{n!} = (-10)\cdot\frac{(-10)\cdot(-10)\cdot\ldots\cdot(-10)(-10)(-10)\cdot\ldots\cdot(-10)(-10)}{1\cdot 2\cdot\ldots\cdot 9\cdot 10\cdot 11\cdot\ldots\cdot(n-1)(n)}$$

$$= (-1)^{n+1}(10)\cdot\underbrace{\frac{10}{1}\cdot\frac{10}{2}\cdot\frac{10}{3}\cdot\ldots\cdot\frac{10}{9}\cdot\frac{10}{10}}_{\geq 1}\cdot\underbrace{\frac{10}{11}\cdot\ldots\cdot\frac{10}{n-1}}_{<1}\cdot\frac{10}{n}$$

$$\therefore\ b_n = -\frac{10^{10}}{10!}\cdot\frac{10}{n} \leq a_n \leq \frac{10^{10}}{10!}\cdot\frac{10}{n} = c_n$$

$$\lim_{n\to\infty} b_n = \lim_{n\to\infty} c_n = 0$$

$\therefore\ \lim_{n\to\infty} a_n = 0$ BY SQUEEZE THEOREM.

Question 2. *(5 marks)* Determine if the following series converges or not. If it does find the sum.

$$\sum_{n=1}^{\infty} \frac{1}{n(n+3)}$$

$$\frac{1}{n(n+3)} = \frac{A}{n} + \frac{B}{n+3} \Rightarrow 1 = A(n+3) + Bn$$

$$n=0 \Rightarrow A = 1/3, \quad n = -3 \Rightarrow B = -1/3$$

$$\therefore \sum_{n=1}^{4} \frac{1}{n(n+3)} = \frac{1}{3}\sum_{n=1}^{\infty}\left(\frac{1}{n} - \frac{1}{n+3}\right)$$

PARTIAL SUM

$$S_n = \left(\frac{1}{1} - \cancel{\frac{1}{4}}\right) + \left(\frac{1}{2} - \cancel{\frac{1}{5}}\right) + \left(\frac{1}{3} - \cancel{\frac{1}{6}}\right) + \left(\cancel{\frac{1}{4}} - \cancel{\frac{1}{7}}\right)$$

$$+ \ldots + \left(\cancel{\frac{1}{n-3}} - \cancel{\frac{1}{n}}\right) + \left(\cancel{\frac{1}{n-2}} - \frac{1}{n+1}\right) + \left(\cancel{\frac{1}{n-1}} - \frac{1}{n+2}\right)$$

$$+ \left(\cancel{\frac{1}{n}} - \frac{1}{n+3}\right)$$

$$= 1 + \frac{1}{2} + \frac{1}{3} - \frac{1}{n+1} - \frac{1}{n+2} - \frac{1}{n+3}$$

$$\therefore \lim_{n\to\infty} S_n = 1 + \frac{1}{2} + \frac{1}{3} = \frac{11}{6}$$

$$\therefore \sum_{n=1}^{\infty} \frac{1}{n(n+3)} = \frac{11}{6}$$

Question 3. *(5 marks)* Use a series to express the repeating decimal 4.17326326326.... as a fraction.

$$4.17\overline{326} = 4.13 + 0.00\overline{326}$$

$$0.00\overline{326} = \frac{326}{100\,000} + \frac{326}{100000000} + \frac{326}{100000000000} + \cdots$$

$$= \frac{326}{100 \cdot 10^3} + \frac{326}{100(10^6)} + \frac{326}{100(10^9)} + \cdots$$

$$= \sum_{n=1}^{\infty} \frac{326}{100(10^3)^n}$$

$$= \sum_{n=1}^{\infty} \frac{326}{100 \cdot 10^3 (10^3)^{n-1}} = \sum_{n=1}^{\infty} \frac{326}{10^5} \cdot \left(\frac{1}{10^3}\right)^{n-1}$$

(GEOMETRIC, $|r| = \frac{1}{10^3} < 1$ $\therefore$ CONVERGES)

$$= \frac{\frac{326}{10^5}}{1 - \frac{1}{10^3}} = \frac{326}{100000} \cdot \frac{1000}{999} = \frac{326}{99900}$$

$$\therefore \; 4.17\overline{326} = \frac{417}{100} + \frac{326}{99900} = \frac{416909}{99900}$$