

Last Name: SOLUTIONS

First Name: \_\_\_\_\_

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## Quiz 11

Question 1. (5 marks) Determine whether the sequence is convergent or not.

$$a_n = \frac{(-10)^{n+1}}{n!} = (-10) \cdot \frac{(-10) \cdot (-10) \cdot \dots \cdot (-10)(-10)(-10) \cdot \dots \cdot (-10)(-10)}{1 \cdot 2 \cdot \dots \cdot 9 \cdot 10 \cdot 11 \cdot \dots \cdot (n-1)(n)}$$

$$= (-1)^{n+1} (10) \cdot \underbrace{\frac{10}{1} \cdot \frac{10}{2} \cdot \frac{10}{3} \cdot \dots \cdot \frac{10}{9} \cdot \frac{10}{10}}_{\geq 1} \cdot \underbrace{\frac{10}{11} \cdot \dots \cdot \frac{10}{n-1} \cdot \frac{10}{n}}_{< 1}$$

$$\therefore b_n = -\frac{10^{10}}{10!} \cdot \frac{10}{n} \leq a_n \leq \frac{10^{10}}{10!} \cdot \frac{10}{n} = c_n$$

$$\lim_{n \rightarrow \infty} b_n = \lim_{n \rightarrow \infty} c_n = 0$$

$\therefore \lim_{n \rightarrow \infty} a_n = 0$  BY SQUEEZE THEOREM.

Question 2. (5 marks) Determine if the following series converges or not. If it does find the sum.

$$\sum_{n=1}^{\infty} \frac{1}{n(n+3)}$$

$$\frac{1}{n(n+3)} = \frac{A}{n} + \frac{B}{n+3} \Rightarrow 1 = A(n+3) + Bn$$

$$n=0 \Rightarrow A = \frac{1}{3}, \quad n = -3 \Rightarrow B = -\frac{1}{3}$$

$$\therefore \sum_{n=1}^{\infty} \frac{1}{n(n+3)} = \frac{1}{3} \sum_{n=1}^{\infty} \left( \frac{1}{n} - \frac{1}{n+3} \right)$$

PARTIAL SUM

$$S_n = \left( \frac{1}{1} - \frac{1}{4} \right) + \left( \frac{1}{2} - \frac{1}{5} \right) + \left( \frac{1}{3} - \frac{1}{6} \right) + \left( \frac{1}{4} - \frac{1}{7} \right)$$

$$+ \dots + \left( \frac{1}{n-3} - \frac{1}{n} \right) + \left( \frac{1}{n-2} - \frac{1}{n+1} \right) + \left( \frac{1}{n-1} - \frac{1}{n+2} \right) \\ + \left( \frac{1}{n} - \frac{1}{n+3} \right)$$

$$= 1 + \frac{1}{2} + \frac{1}{3} - \frac{1}{n+1} - \frac{1}{n+2} - \frac{1}{n+3}$$

$$\therefore \lim_{n \rightarrow \infty} S_n = 1 + \frac{1}{2} + \frac{1}{3} = \frac{11}{6}$$

$$\therefore \sum_{n=1}^{\infty} \frac{1}{n(n+3)} = \frac{1}{3} \left( \frac{11}{6} \right) = \frac{11}{24}$$

Question 3. (5 marks) Use a series to express the repeating decimal  $4.17\overline{326}$  as a fraction.

$$4.17\overline{326} = 4.13 + 0.00\overline{326}$$

$$0.00\overline{326} = \frac{326}{100000} + \frac{326}{10000000} + \frac{326}{10000000000} + \dots$$

$$= \frac{326}{100 \cdot 10^3} + \frac{326}{100(10^6)} + \frac{326}{100(10^9)} + \dots$$

$$= \sum_{n=1}^{\infty} \frac{326}{100(10^3)^n}$$

$$= \sum_{n=1}^{\infty} \frac{326}{100 \cdot 10^3 (10^3)^{n-1}} = \sum_{n=1}^{\infty} \frac{326}{10^5} \cdot \left(\frac{1}{10^3}\right)^{n-1}$$

(GEOMETRIC,  $|r| = \frac{1}{10^3} < 1 \quad \therefore$  CONVERGES)

$$= \frac{\frac{326}{10^5}}{1 - \frac{1}{10^3}} = \frac{326}{100000} \cdot \frac{1000}{999} = \frac{326}{99900}$$

$$\therefore 4.17\overline{326} = \frac{417}{100} + \frac{326}{99900} = \frac{416909}{99900}$$