

Last Name: SOLUTIONS

First Name: _____

Student ID: _____

Quiz 2

Question 1. (2 marks) Express the following in sigma notation:

$$3 - \frac{4}{2} + \frac{5}{4} - \frac{6}{8} + \frac{7}{16} - \frac{8}{32} = \sum_{i=1}^6 \frac{i+2}{2^{i-1}} (-1)^{i+1}$$

Question 2. (3 marks) Find the value of the following sum:

$$\begin{aligned} \sum_{i=1}^{1000} i(i-3) &= \sum_{i=1}^{1000} (i^2 - 3i) = \sum_{i=1}^{1000} i^2 - 3 \sum_{i=1}^{1000} i \\ &= \frac{1000(1000+1)(2 \cdot 1000 + 1)}{6} - 3 \frac{(1000)(1000+1)}{2} \\ &= 333833500 - 1501500 \\ &= 332332000 \end{aligned}$$

Question 3. (5 marks) Estimate the area under the graph of $f(x) = \cos x$ from $x = 0$ to $x = \pi/2$ using four approximating rectangles and right endpoints (you may use decimals). Repeat using left endpoints (indicate which approximation is which).

$$\Delta x = \frac{\pi/2 - 0}{4} = \pi/8 \quad \therefore x_0 = 0, x_1 = \pi/8, x_2 = \pi/4, x_3 = 3\pi/8, x_4 = \pi/2$$

$$\begin{aligned} \therefore R_4 &= f(x_1)\Delta x + f(x_2)\Delta x + f(x_3)\Delta x + f(x_4)\Delta x \\ &= \cos(\pi/8) \cdot \pi/8 + \cos(\pi/4) \cdot \pi/8 + \cos(3\pi/8) \cdot \pi/8 + \cos(\pi/2) \cdot \pi/8 \\ &= 0.79077 \end{aligned}$$

$$\begin{aligned} L_4 &= f(x_0)\Delta x + f(x_1)\Delta x + f(x_2)\Delta x + f(x_3)\Delta x \\ &= \cos(0) \cdot \pi/8 + \cos(\pi/8) \cdot \pi/8 + \cos(\pi/4) \cdot \pi/8 + \cos(3\pi/8) \cdot \pi/8 \\ &= 1.18347 \end{aligned}$$