February 6, 2013

Last Name: SOLUTIONS

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Quiz 3

Question 1. *(10 marks)* Use the limit definition of the definite integral to evaluate

$$\int_1^4 (x^2-4x+2)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

- $\Delta x = \frac{b-a}{n} = \frac{4-1}{n} = \frac{3}{n}$, $x_i = a + i\Delta x = 1 + \frac{3i}{n}$
- $f(x_i) = x_i^2 - 4x_i + 2 = \left(1+\frac{3i}{n}\right)^2 - 4\left(1+\frac{3i}{n}\right) + 2$

$$= 1 + \frac{6i}{n} + \frac{9i^2}{n^2} - 4 - \frac{12i}{n} + 2 = -1 - \frac{6i}{n} + \frac{9i^2}{n^2}$$

- $f(x_i)\Delta x = \left(-1 - \frac{6i}{n} + \frac{9i^2}{n^2}\right)\left(\frac{3}{n}\right) = -\frac{3}{n} - \frac{18i}{n^2} + \frac{27i^2}{n^3}$
- $\sum_{i=1}^{n} f(x_i)\Delta x = \sum_{i=1}^{n}\left(-\frac{3}{n} - \frac{18i}{n^2} + \frac{27i^2}{n^3}\right) = -\frac{3}{n}\sum_{i=1}^{n} 1 - \frac{18}{n^2}\sum_{i=1}^{n} i + \frac{27}{n^3}\sum_{i=1}^{n} i^2$

$$= -\frac{3}{n}\cdot n - \frac{18}{n^2}\cdot\frac{n(n+1)}{2} + \frac{27}{n^3}\cdot\frac{n(n+1)(2n+1)}{6}$$

$$= -3 - 9\cdot\frac{n+1}{n} + \frac{9}{2}\cdot\frac{2n^2+3n+1}{n^2}$$

$$\therefore \int_1^4 x^2 - 4x + 2\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

$$= \lim_{n\to\infty}\left[-3 - 9\cdot\frac{n+1}{n} + \frac{9}{2}\cdot\frac{2n^2+3n+1}{n^2}\right]$$

$$= -3 - 9\cdot 1 + \frac{9}{2}\cdot 2$$

$$= -3 - 9 + 9$$

$$= -3$$