Dawson College: Calculus II: 201-NYB-05

February 20, 2013

Last Name: SOLUTIONS

First Name: _____

Student ID: _____

Quiz 5

Question 1. *(3 marks)* Find the average value of $f(\theta) = \sec\theta\tan\theta$ on the interval $[0, \pi/4]$

$$f_{ave} = \frac{1}{b-a}\int_a^b f(\theta)\,d\theta = \frac{1}{\frac{\pi}{4}-0}\int_0^{\pi/4} \sec\theta\tan\theta\,d\theta$$

$$= \frac{4}{\pi}\left[\sec\theta\right]_0^{\pi/4} = \frac{4}{\pi}\left[\sec\pi/4 - \sec 0\right]$$

$$= \frac{4}{\pi}\left[\frac{1}{\cos\pi/4} - \frac{1}{\cos 0}\right] = \frac{4}{\pi}\left[\frac{1}{\sqrt{2}/2} - \frac{1}{1}\right] = \frac{4}{\pi}\left(\frac{2}{\sqrt{2}} - 1\right)$$

Question 2. Evaluate the following definite integrals

(a) *(3 marks)* $\int_{-\pi/2}^{\pi/2} \frac{x^2\sin x}{1+x^6}dx$

$\sin x$ IS ODD

LET $f(x) = \frac{x^2\sin x}{1+x^6} \Rightarrow f(-x) = \frac{(-x)^2\sin(-x)}{1+(-x)^6} = \frac{x^2(-\sin x)}{1+x^6}$

$$= -\frac{x^2\sin x}{1+x^6} = -f(x)$$

$\therefore$ f IS ODD

$$\therefore \int_{-\pi/2}^{\pi/2} \frac{x^2\sin x}{1+x^6}dx = 0$$

(b) *(4 marks)* $\int_e^{e^4} \frac{dx}{x\sqrt{\ln x}}$

LET $u = \ln x$

$du = \frac{1}{x}\,dx$

$x\,du = dx$

IF $x = e \Rightarrow u = \ln e = 1$

$x = e^4 \Rightarrow u = \ln e^4 = 4\ln e = 4$

$$= \int_1^4 \frac{x\,du}{x\sqrt{u}}$$

$$= \int_1^4 \frac{du}{u^{1/2}} = \int_1^4 u^{-1/2}\,du$$

$$= 2u^{1/2}\Big|_1^4 = 2(4)^{1/2} - 2(1) = 2\cdot 2 - 2 = 2$$