Dawson College: Calculus II: 201-NYB-05 March 6, 2013

Last Name: SOLUTIONS

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Quiz 6

Question 1. Find the following

(a) *(5 marks)* $\int \cos^4(3t)\sin^2(3t)dt = \int \left(\frac{1+\cos 6t}{2}\right)^2\left(\frac{1-\cos 6t}{2}\right)dt$

$= \frac{1}{8}\int (1+2\cos 6t+\cos^2 6t)(1-\cos 6t)dt$

$= \frac{1}{8}\int 1+2\cos 6t+\cos^2(6t)-\cos 6t-2\cos^2 6t-\cos^3 6t\,dt$

$= \frac{1}{8}\int 1+\cos 6t-\left(\frac{1+\cos 12t}{2}\right)-(1-\sin^2 6t)\cos 6t\,dt$

$= \frac{1}{8}\int \frac{1}{2}+\cos 6t-\frac{1}{2}\cos 12t\,dt \;-\frac{1}{8}\int (1-\sin^2 6t)\cos 6t\,dt$

LET $u=\sin 6t$

$du = 6\cos 6t\,dt$

$= \frac{1}{8}\left[\frac{1}{2}t+\frac{1}{6}\sin 6t-\frac{1}{2}\cdot\frac{1}{12}\sin 12t\right]-\frac{1}{8}\cdot\frac{1}{6}\int (1-u^2)du$

$= \frac{1}{16}t+\frac{1}{48}\sin 6t-\frac{1}{192}\sin 12t-\frac{1}{48}\left[u-\frac{u^3}{3}\right]+C$

$= \frac{1}{16}t+\frac{1}{48}\sin 6t-\frac{1}{192}\sin 12t-\frac{1}{48}\sin 6t+\frac{1}{144}\sin^3 6t+C$

$= \frac{1}{16}t-\frac{1}{192}\sin 12t+\frac{1}{144}\sin^3 6t+C$

(b) *(5 marks)* $\int_{\pi/6}^{\pi/4} \cot^5 x \csc^3 x\,dx = \int_{\pi/6}^{\pi/4} \cot^4 x \csc^2 x \cot x \csc x\,dx$

LET $u = \csc x$

$du = -\csc x \cot x\,dx$

IF $x = \pi/6$, $u = 2$

$x = \pi/4$, $u = 2/\sqrt{2} = \sqrt{2}$

$$= \int_{\pi/6}^{\pi/4} (\csc^2 x - 1)^2 \csc^2 x \cot x \csc x\,dx$$

$$= -\int_{\sqrt{2}}^{2} (u^2 - 1)^2 u^2\,du$$

$$= -\int_{\sqrt{2}}^{2} u^6 - 2u^4 + u^2\,du$$

$$= -\left[\frac{1}{7}u^7 - \frac{2}{5}u^5 + \frac{1}{3}u^3\right]_{\sqrt{2}}^{2}$$

$$= -\left[\left(\frac{128}{7} - \frac{64}{5} + \frac{8}{3}\right) - \left(\frac{8\sqrt{2}}{7} - \frac{8\sqrt{2}}{5} + \frac{2\sqrt{2}}{3}\right)\right]$$

$$= -\frac{856}{105} + \frac{22\sqrt{2}}{105}$$

(c) (5 marks) $\int \frac{dx}{\sqrt{x^2-4x}}$

$x^2-4x+4-4 = (x-2)^2-4$

LET $u = x-2$

$du = dx$

LET $u = 2\sec\theta$ ON $0 \le \theta < \pi/2$

$\pi \le \theta < 3\pi/2$

$du = 2\sec\theta\tan\theta\, d\theta$

$$= \int \frac{dx}{\sqrt{(x-2)^2-4}} = \int \frac{du}{\sqrt{u^2-4}} =$$

$$= \int \frac{2\sec\theta\tan\theta\, d\theta}{2\sqrt{\sec^2\theta-1}} = \int \frac{\sec\theta\tan\theta\, d\theta}{\sqrt{\tan^2\theta}} = \int \frac{\sec\theta\tan\theta}{\tan\theta}\, d\theta$$

SINCE $\tan\theta \ge 0$ ON SPECIFIED INTERVAL

$$= \int \sec\theta\, d\theta = \ln|\sec\theta + \tan\theta| + C$$

$$= \ln\left|\frac{u}{2} + \frac{\sqrt{u^2-4}}{2}\right| + c$$

$$= \ln\left|\frac{x-2}{2} + \frac{\sqrt{x^2-4x}}{2}\right| + C$$

$\frac{u}{2} = \sec\theta$