

Last Name: SOLUTION

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## Quiz 7

Question 1. Find the following

(a) (5 marks)  $\int \frac{x^3 - 4x - 10}{x^2 - x - 6} dx = I$

$$\therefore I = \int x + 1 + \frac{3x - 4}{(x - 3)(x + 2)} dx$$

$$\begin{array}{r} x + 1 \\ \hline x^2 - x - 6 \overline{) x^3 + 0x^2 - 4x - 10} \\ \underline{-(x^3 - x^2 - 6x)} \phantom{-10} \\ x^2 + 2x - 10 \\ \underline{-(x^2 - x - 6)} \\ 3x - 4 \end{array}$$

$$\frac{3x - 4}{(x - 3)(x + 2)} = \frac{A}{x - 3} + \frac{B}{x + 2}$$

$$3x - 4 = A(x + 2) + B(x - 3)$$

IF  $x = 3$

$$5A = 5 \Rightarrow A = 1$$

IF  $x = -2$

$$-10 = -5B \Rightarrow B = 2$$

$$\therefore I = \int x + 1 + \frac{1}{x - 3} + \frac{2}{x + 2} dx = \frac{x^2}{2} + x + \ln|x - 3| + 2\ln|x + 2| + C$$

$$(b) \text{ (5 marks)} \int \frac{x+4}{x^2+2x+5} dx = \underbrace{\int \frac{x+1}{x^2+2x+5} dx}_{I_1} + \underbrace{\int \frac{3}{x^2+2x+5} dx}_{I_2} = I$$

$$\begin{aligned} \text{LET } u &= x^2+2x+5 \\ du &= 2x+2 dx \\ du &= 2(x+1) dx \end{aligned} \Rightarrow I_1 = \int \frac{x+1}{u} \frac{du}{2(x+1)} = \frac{1}{2} \ln|x^2+2x+5| + C$$

$$I_2 = \int \frac{3}{(x+1)^2+4} = 3 \int \frac{1}{u^2+4} du = \frac{3}{4} \int \frac{1}{\left(\frac{u}{2}\right)^2+1} du$$

$$\boxed{\begin{array}{l} \text{LET } u = x+1 \\ du = dx \end{array}} = \frac{3}{4} \cdot 2 \arctan\left(\frac{u}{2}\right) + C$$

$$= \frac{3}{2} \arctan\left(\frac{x+1}{2}\right) + C$$

$$\begin{aligned} \therefore I &= I_1 + I_2 \\ &= \frac{1}{2} \ln|x^2+2x+5| + \frac{3}{2} \arctan\left(\frac{x+1}{2}\right) + C \end{aligned}$$