Dawson College: Calculus II: 201-NYB-05

March 26, 2013

Last Name: SOLUTIONS

First Name: __________

Student ID: __________

Quiz 8

Question 1. *(5 marks)* Evaluate the integral or show that it is divergent

$$I = \int_1^\infty \frac{\arctan x}{x^2}\,dx = \lim_{t\to\infty}\int_1^t \frac{\arctan x}{x^2}\,dx$$

$$\int \frac{\arctan x}{x^2}\,dx = \underbrace{-\frac{\arctan x}{x}}_{\text{INTEGRATION BY PARTS}} + \int \frac{1}{x}\cdot\frac{dx}{1+x^2} = -\frac{\arctan x}{x} + \underbrace{\int\left[\frac{1}{x} - \frac{x}{x^2+1}\right]dx}_{\text{PARTIAL FRACTIONS}}$$

LET $u = \arctan x \qquad dv = \frac{1}{x}\,dx$

$du = \frac{1}{1+x^2}\,dx \qquad V = -\frac{1}{x^2}$

$$= -\frac{\arctan x}{x} + \ln|x| - \frac{1}{2}\ln(x^2+1) + C = \frac{\arctan x}{x} + \frac{1}{2}\ln\frac{x^2}{x^2+1} + C$$

$$\therefore I = \lim_{t\to\infty}\left[-\frac{\arctan x}{x} + \frac{1}{2}\ln\left(\frac{x^2}{x^2+1}\right)\right]_1^t$$

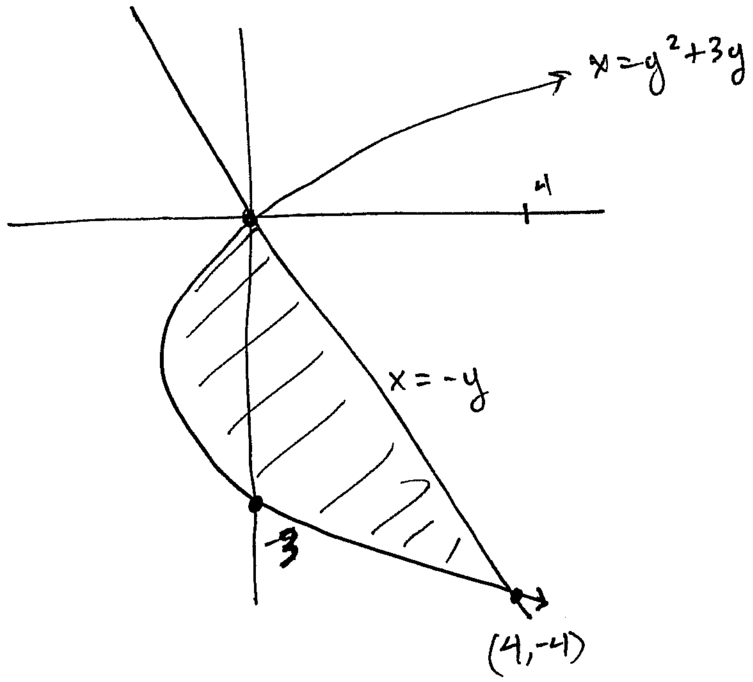
$$= \lim_{t\to\infty}\left[-\frac{\overset{\pi/2}{\arctan t}}{\underset{\to\infty}{t}} + \frac{1}{2}\ln\frac{t^2}{t^2+1} + \frac{\arctan(1)}{(1)} - \frac{1}{2}\ln\left(\frac{1^2}{1^2+1}\right)\right]$$

$$= 0 + \frac{1}{2}\ln(1) + \frac{\pi}{4} - \frac{1}{2}\ln(1/2)$$

$$= \pi/4 - \frac{1}{2}\ln(1/2)$$

Question 2. *(5 marks)* Find the area bounded by $x+y=0$ and $x=y^2+3y$.

$y^2+3y=-y \iff y^2+4y=0 \iff y(y+4)=0 \iff y=0,-4$



$$\therefore\ A=\int_{-4}^{0}(-y)-(y^2+3y)\,dy$$

$$=\int_{-4}^{0}(-y^2-4y)\,dy$$

$$=\left[-\tfrac{1}{3}y^3-2y^2\right]_{-4}^{0}=0-\left(\frac{64}{3}-32\right)$$

$$=\frac{32}{3}$$