

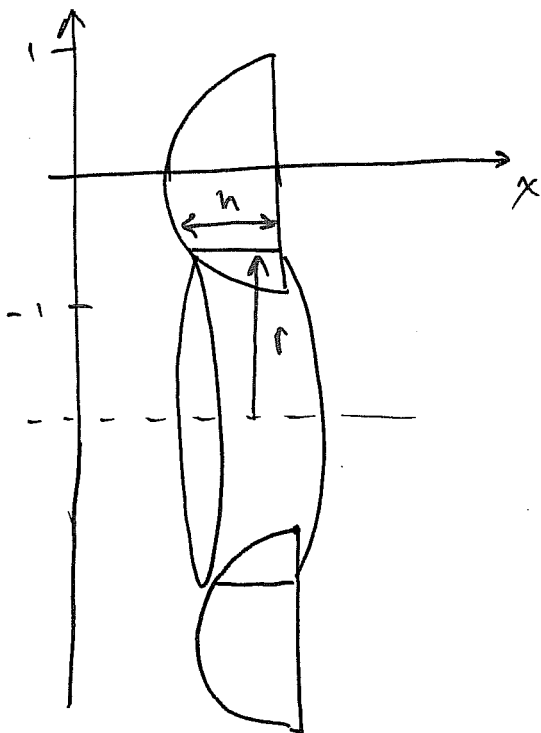
Last Name: SOLUTIONS

First Name: _____

Student ID: _____

Quiz 9

Question 1. (5 marks) Use the method of cylindrical shells to find the volume generated by rotating the region bounded by the curves $x = y^2 + 1$, $x = 2$ about $y = -2$.



$$\text{shell: } h = 2 - (y^2 + 1) = 1 - y^2$$

$$r = 2 + y$$

$$\begin{aligned} A(y) &= 2\pi r h = 2\pi (2 + y)(2 - (y^2 + 1)) \\ &= 2\pi (2 + y)(1 - y^2) \end{aligned}$$

$$V = \int_{-1}^1 2\pi (y - y^3 + 2 - 2y^2) dy$$

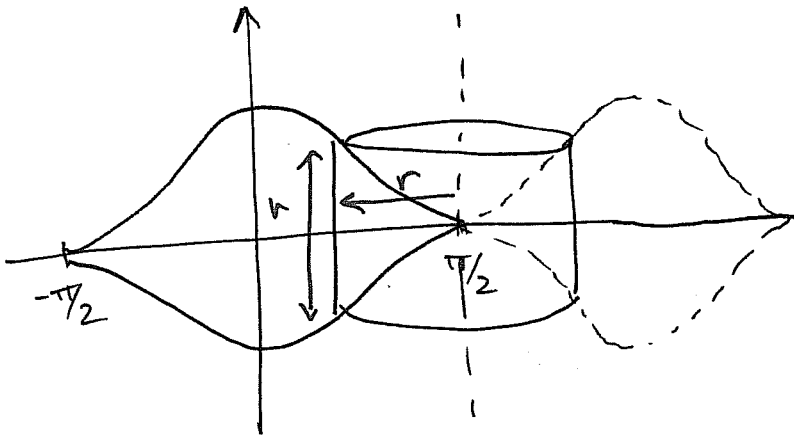
$$= 2\pi \left[\frac{1}{2}y^2 - \frac{1}{4}y^4 + 2y - \frac{2}{3}y^3 \right]_{-1}^1$$

$$= 2\pi \left[\left(\frac{1}{2} - \frac{1}{4} + 2 - \frac{1}{3} \right) - \left(\frac{1}{2} - \frac{1}{4} - 2 + \frac{2}{3} \right) \right]$$

$$= \frac{16}{3}\pi \text{ units}^3$$

Question 2. (5 marks) Sketch the graph of the region bounded between the graphs of $y = \cos^4 x$ and $y = -\cos^4 x$ on the interval $-\pi/2 \leq x \leq \pi/2$.

Set up an integral that gives the volume of the solid generated by rotating this region about the line $x = \pi/2$. (Add the appropriate information to the graph of the region).



$$\text{Shell: } h = \cos^4 x - (-\cos^4 x) = 2\cos^4 x$$

$$r = \pi/2 - x$$

$$\therefore A(x) = 2\pi r h = 2\pi (\pi/2 - x) \cdot 2\cos^4 x$$

$$\therefore V = \int_{-\pi/2}^{\pi/2} 4\pi (\pi/2 - x) \cos^4 x \, dx$$