Continuous Random Variables

We have seen a few examples of probability distributions so far but the single most important probability distribution to statisticians is probably the **normal probability distribution.**

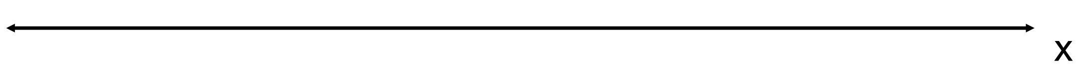
The normal probability distribution has a continuous random variable and uses two functions: one to determine the y-values of the graph of the distribution and a second to determine the probabilities.

Normal Probability Distribution Function

$$y = f(x) = \frac{e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}}{\sigma\sqrt{2\pi}}$$

for all x.

when this is graphed we get a bell shaped curve which is called the normal curve.



We will get a different function for each value of μ (mean) and σ (standard deviation).

To get the probability function for this distribution we look at the definite integral of f(x). The probability that x is within the interval x=a to x= b we find

$$P(a \leq x \leq b) = \int_a^b f(x)dx$$

There are infinitely many normal probability distributions but we can relate them all to the **standard normal distribution**.

Properties of the standard normal distribution

Finding the area beneath the standard normal curve

Since we are unable to compute the antiderivative $\displaystyle\int \frac{e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}}{\sigma\sqrt{2\pi}}dx$

analytically we can't use the fundamental theorem of calculus to find

$$P(a \leq x \leq b) = \int_a^b \frac{e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}}{\sigma\sqrt{2\pi}}dx$$

Instead we approximate these integrals numerically and generate a table of probabilities (areas under the curve). Thankfully this has been done for us.

Reading a table to find probabilities:

The table gives the values of areas on the right side of the curve starting from 0.

z	0.00	0.01	0.02	0.03
0.0				
0.1				
0.2				
0.3				
0.4				

Reading the table in this way we can compute probabilities. For example, if we want

$$P(0.00 < z < 1.52)$$

we would read from the table

z	0.00	0.01	0.02	0.03
⋮				
1.5				
⋮				

The structor of the standard normal distribution allows us to calculate other probabilities:

<u>Right tail of a normal curve</u>

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Left of a positive z-value

From a negative z-value to z=0

Between two z-values of the same sign

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Examples:

1) What is the z-score associated with the 75th percentile of normal distribution?

2) Which z-scores bound the middle 95% of a normal distribution?

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3) Consider IQ scores. They are normally distributed with a mean of 100 and standard deviation of 16. What is the probability that a person selected at random will have an IQ between 100 and 115?

4) Find the probability that a person selected at random will have an IQ greater than 90.

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Example: Pronto pizza guarantees that your pizza will be delivered within 45 minutes or it's free. They would like to reduce the guaranteed delivery time. What guaranteed delivery time would result in at most 5% of pizzas being free? Assume that delivery times are normally distributed with mean = 30min and standard deviation = 6.1min.

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