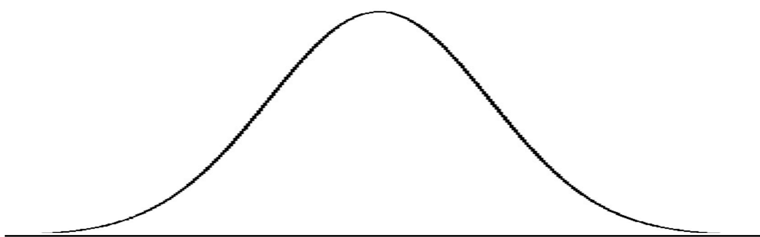
Critical Values of z

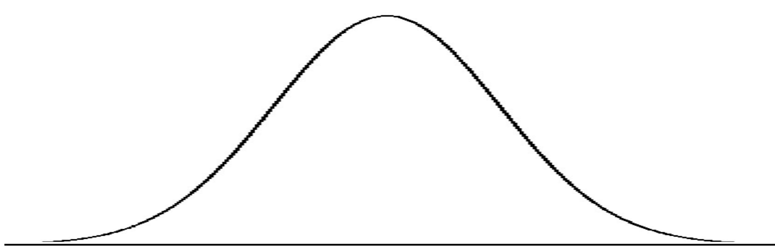
Some notation:

We will let $z(\alpha)$ (sometimes written z_α) be the z-score such that an area of α lies to the right of $z(\alpha)$.

For example, if $\alpha = 0.05$ then we have the following situation



$\alpha/2$ will refer to the area at both tails of the normal curve. For example $\alpha/2 = 0.025$ would be



$z(\alpha/2)$ is the positive z-score associated with $\alpha/2$. In the above case

Example: find $z(\alpha)$ for $\alpha = 0.15$. Find $z(\alpha/2)$ for $\alpha/2 = 0.15$.
Draw the normal curve and area for each case.

The values $z(\alpha)$ and $z(\alpha/2)$ corresponding to tail areas α and $\alpha/2$ are called **critical values of z**.

The following critical values used most often are

α or $\alpha/2$ (area)	0.1	0.05	0.025	0.01	0.005
$z(\alpha)$ or $z(\alpha/2)$	1.28	1.645	1.96	2.33	2.575

We can see that the normal distribution is very useful but even if we know if a population is normally distributed we still need to know the mean μ and the standard deviation σ for the population. Unless we do a census of every member of the population we will have to rely on samples.

What is the relationship between a sample mean $\overline{x}$ and the population mean μ?

To answer this question we will look at another probability distribution called the sampling distribution.

Sampling Distribution and Sample Mean

Consider a certain population called the **parent population** P_x from which we draw random samples of size n.

With this sample of size n we can compute the sample mean $\overline{x}$. If we repeat this (taking a sample of size n) one thousand times we will get one thousand corresponding $\overline{x}$ values. In this way we can consider $\overline{x}$ to be a random variable.

The collection of all possible $\overline{x}$ values form a new population $P_{\overline{x}}$. (Note that there is a different $\overline{x}$ for each sample size n)

Let's consider the finite population {0,2,4,6,8}. In this case the population mean is $\mu = 5$. Now let's consider all possible samples of size 2:

{0,0}	{2,0}	{4,0}	{6,0}	{8,0}
{0,2}	{2,2}	{4,2}	{6,2}	{8,2}
{0,4}	{2,4}	{4,4}	{6,4}	{8,4}
{0,6}	{2,6}	{4,6}	{6,6}	{8,6}
{0,8}	{2,8}	{4,8}	{6,8}	{8,8}

For each sample the mean $\overline{x}$ is

0	1	2	3	4
1	2	3	4	5
2	3	4	5	6
3	4	5	6	7
4	5	6	7	8

Each of these samples is equally likely so we can make the probability distribution for $\overline{x}$

$\overline{x}$ $\quad$ $P(\overline{x})$

The mean for this distribution is $\mu_{\overline{x}}$

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Let $\sigma_{\overline{x}}$ be the standard deviation of the random variable $\overline{x}$. Then we can calculate

where n is the sample size. This is assuming we sample with replacement. If we sample a finite population without replacement then we must multiply by the **finite population correction factor**

We will only use the finite population correction factor when the sample size n is more than 5% of the total population N (which is not very often for us). Otherwise we see that

The mean for all possible $\overline{x}$ is actually the same as the mean for x

$$\mu_{\overline{x}} = \mu_x \text{ or } \mu$$

<u>Summary:</u>

Parent Population

Sampling Distribution of Sample Means

Example: Consider a population of infinite size made up of only values x=2,3,4,5,6,7, and 8 with the following distribution

x	P(x)
2	0.256
3	0.077
4	0.077
5	0.103
6	0.051
7	0.154
8	0.282

We can use a computer to generate samples to calculate the probability distributions for $\overline{x}$. We get the following results.

The mean $\mu_{\overline{x}} = \mu$ remains the same but the standard deviation gets smaller as sample size increases. The sampling distribution are becoming more normal.

The Central Limit Theorem

For sampling distributions of sample means of any parent population P, as sample size increases the sampling distributions migrate towards μ, the central value, and they approach the shape of a normal distribution.

What this means:

sample size: n=5

sample size: n=50

For larger sample sizes a sample mean has a better chance of being closer to the population mean.