5.3 Evaluating Definite Integrals

We now have two notations that look similar but mean different things. We have to be careful not to confuse the two.

$\int_a^b f(x)\mathrm{d}x$	$\int f(x)\mathrm{d}x$
• this is a definite integral • you are finding the net area • the final answer should be a number!	• this is the indefinite integral • you are finding all antiderivatives of $f(x)$ • the final answer is function!

Example:

a) Use the limit definition of the definite integral to evaluate $\int_{-1}^{2}(3x^2-2x+1)\mathrm{d}x$

b) Find the following antiderivative $F(x)=\int(3x^2-2x+1)\mathrm{d}x$

c) Just for fun, find $F(-1)$ and $F(2)$.

Evaluation Theorem (the Fundamental Theorem of Calculus part 2)

If f is a continuous function on the interval $[a,b]$ then

where F is any antiderivative of f.

First let's see how it works, then let's look at why it works.

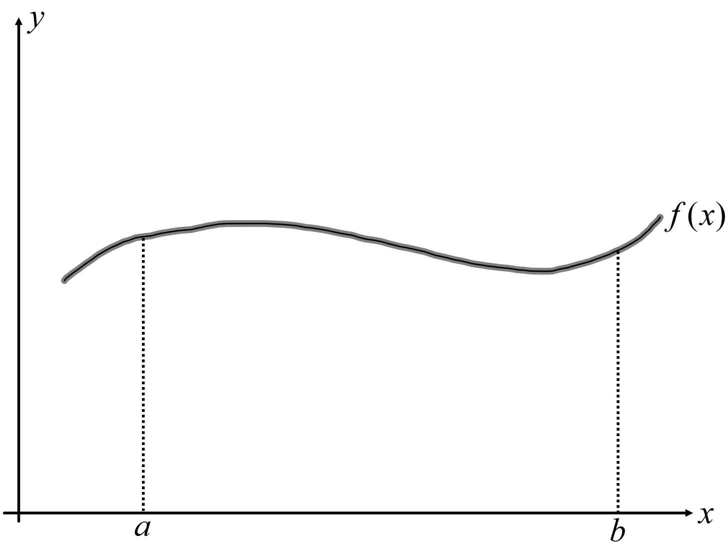
Example: Evaluate $\int_0^1 \frac{1}{x^2}\mathrm{d}x$

To see why it works we need to define a new function.

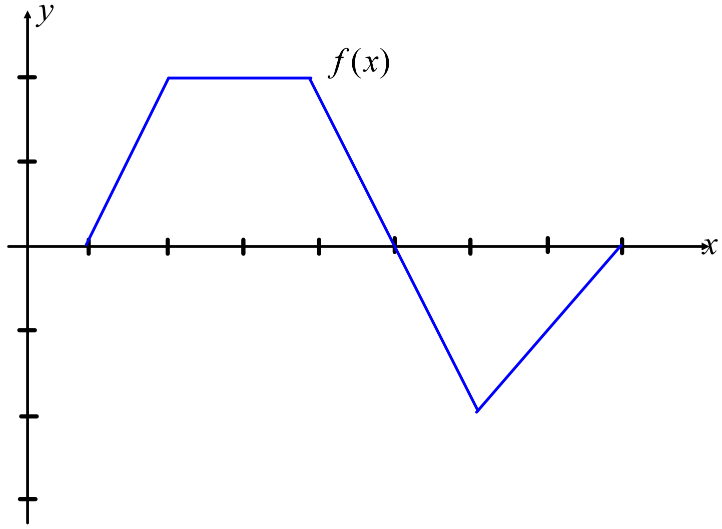
Let $f(x)$ be a continuous function on the interval $[a,b]$, like in the statement of the theorem. We define the area function, $A(x)$, as follows:

$$A(x) = \int_a^x f(x)\mathrm{d}x \quad \text{for} \quad a \le x \le b$$

So for any x-value that we plug into the function, it tells us the net area from a to x.



Example: Let $f(x)$ be the function below and let $A(x)=\int_1^x f(x)\mathrm{d}x$ for $1\le x\le 8$



a) Find $A(2)$

b) Find $A(5)$

c) Find $A(6)$

d) Find $A(8)$

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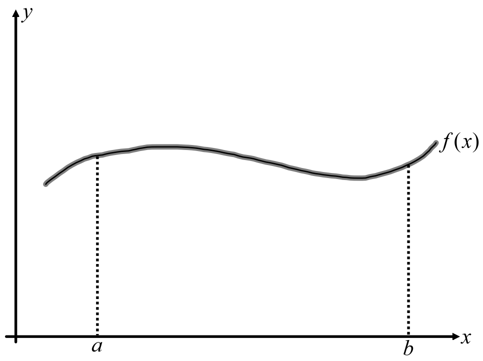
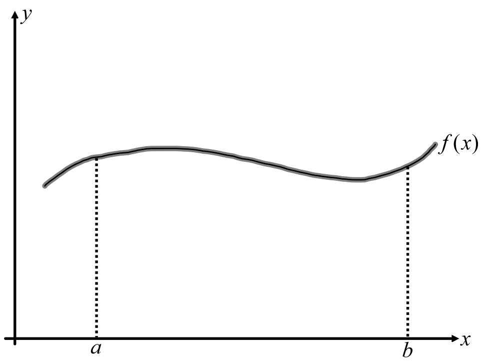
Back to the more general situation:

$f(x)$ is a continuous function on the interval $[a,b]$ and

$$A(x) = \int_a^x f(x)\mathrm{d}x \quad \text{for} \quad a \le x \le b$$

Let's look at a small strip of area





When applying the evaluation theorem we use the notation

$F(x)\big|_a^b = F(b) - F(a)$ or $\big[F(x)\big]_a^b = F(b) - F(a)$

Examples: Evaluate the following definite integrals.

a) $\int_0^{\pi/3} \cos(\theta)\mathrm{d}\theta$

b) $\int_1^4 \frac{(x-4)(x-2)}{x^2}$ dx

c) $\int_{\pi/4}^{\pi/3} \tan^2\theta \ \mathrm{d}\theta$

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d) $\int_1^9 \frac{2t^2 + t^2\sqrt{t} - 1}{t^2}\,\mathrm{d}t$

$$\int_0^{1/2} \left(3x^2 - 4x + \frac{1}{\sqrt{1 - x^2}} \right) \mathrm{d}x$$

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