Hypothesis Testing

Suppose a friend is having a party and you are invited but you are not sure if you should go or not. You think the "party will be a dud" but your friend assures you that it "will be great."

Obviously you would make this decision, as any good statistician would, by formulating two hypotheses. In this case the two hypothesis would be

These are two opposing hypotheses and you know one of them must be true.

The process by which a decision is made between two opposing hypotheses is called a **statistical hypothesis test**.

The two hypotheses involved in making a decision are known as the null hypothesis and the alternative hypothesis.

When deciding which statement should be the null hypothesis and the alternative hypothesis we consider a couple of things.

1) The basic idea of the hypothesis test is for the evidence to have a chance to disprove the null hypothesis. (The null hypothesis can be rejected by the evidence).

2) You are most likely designing the experiment because you think there will be a chance of rejecting the null hypothesis. The null hypothesis is sometimes the research hypothesis since it represents what the researcher hopes will be found to be "true."

In the case of the party, since collecting evidence would seems to prove the unlikeliness of the party being a dud it, it's natural to start by assuming the party will be a dud.

Writing Hypothesis:

Situation: You are testing a new design for airbags used in automobiles, and you are concerned that they might not open properly. What should the null and alternative hypothesis be?

Situation: You suspect that a brand-name detergent outperforms the store's brand of detergent and you wish to test the two detergents because you would prefer to by the cheaper store brand. What should the null and alternative hypothesis be?

As we stated earlier, usually the null hypothesis is a statement that a population parameter has a specific value.

For example:

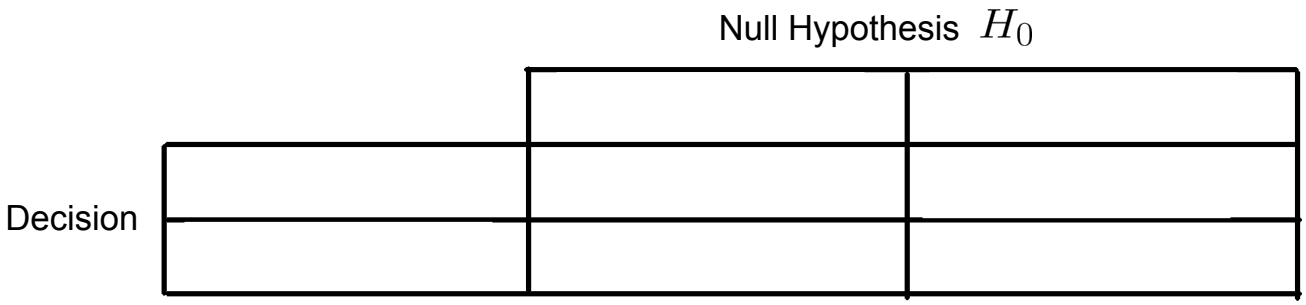
1) The standard brand of latex paint dries in 90 minutes on average. A company claims that their paint dries faster. State the null and alternate hypotheses.

Where μ is the mean dry time dry time for the company's paint.

(Note: We always include the "equality" in the null hypothesis.)

2) It is thought that the mean salary for jobs that fall into a the category of "blue-collar jobs" in montreal is no more than $35 000 per year. You want to create a hypothesis test to test this hypothesis. State the null and alternate hypotheses.

The goal of a hypothesis test is to decided whether or not we should reject the null hypothesis. This means that a hypothesis test can have four outcomes:



The **level of significance** α is the probability of committing a type I error. In other words, the probability of rejecting a true null hypothesis.

We usually want, at most, a 5% chance of rejecting a true H_0, that is

$$\alpha \leq 0.05$$

Making a Decision

When performing a hypothesis test we

- begin by assuming the null hypothesis is true
- evaluate the evidence to determine if it is significant enough to reject H_0

Hypothesis Test of Mean (σ known): A Classical Approach

Suppose we have a null hypothesis about the value of a mean of a population. There are three possible null hypothesis:

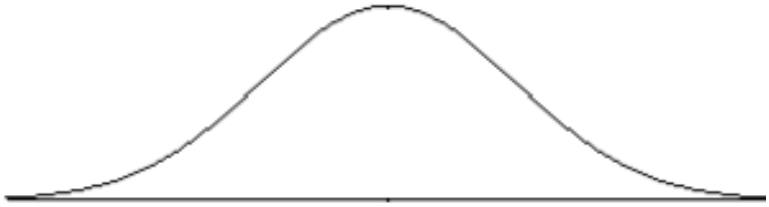
1) $H_0 : \mu \geq \mu_0$
$H_a : \mu < \mu_0$

for example

$H_0 : \mu \geq 90$ min
$H_a : \mu < 90$ min (faster)

We want to make sure that the chances of rejecting a true null hypothesis is less than the level of significance α.

Suppose we sample the population (size n) and calculate the sample mean. Let's look at the probability distribution.



If μ_0 were the correct mean the chances of obtaining a sample mean in the shaded region above would be less than α. Meaning if we obtain a sample mean in the shaded region it is unlikely that $\mu = \mu_0$, in fact it is unlikely that $\mu \geq \mu_0$, in which case we would want to reject the null hypothesis.

For this reason, the shaded region above is called the **rejection region** (or the **critical region**). The non-shaded region is called the **acceptance region** (or the noncritical region).

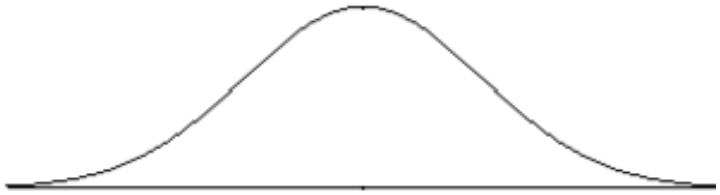
This is called a **left tail test.**

Similarly:

2) If $H_0 : \mu \leq \mu_0$

$H_a : \mu > \mu_0$

we use the **right tail test.**



3) If $H_0 : \mu = \mu_0$

$H_a : \mu \neq \mu_0$