

Classical Approach to Hypothesis Testing

Summary: (test of hypothesis concerning one population)

0 - Describe the random variable x

2 - State H_0 and H_a

3 - Specify α , the level of significance

4 - Justify the use of z-table (either the population has a normal distribution or $n \geq 30$)

5 - Determine rejection region

6 - Calculate the test statistic $z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$

7 - Determine whether to reject H_0

8 - Write a brief conclusion interpreting the decision

Example: Standard brand of latex paint dries in 90 minutes on average. The rival company claims that their latex paint dries faster. A random sample of 34 cans of rival paint gave a mean drying time of 87.1 min. Test the rival company's claim at 5% significance if $\sigma = 8.2$.

Example: A coffee dispenser is designed to fill cups with exactly 250ml of liquid. For obvious reasons, it is not good to pour too much or too little coffee into the cups. Assume that the volumes are normally distributed with $\sigma = 14.5$ ml. Is the machine filling properly if 12 cups were filled with an average volume of 255.4ml? Conduct a test of hypothesis at 10% significance level.

Note: There is a chance that we made a type II error,

$P(\text{Accept } H_0 | H_0 \text{ is actually false})$. This is not, however, the same as α , the probability of a type I error, that is $P(\text{Reject } H_0 | H_0 \text{ is actually true})$.

Hypothesis Test of Mean (σ known): A Classical Approach

The probability of getting a sample result at least as extreme as the one produced by a random sample is called the p-value. In a one-tail test of hypothesis, the p-value is simply the area in the tail determined by the sample mean \bar{x} .

Example: Calculate the p-value in the test of hypothesis for mean drying time for the new latex paint.

The p-value approach

- Reject H_0 if p-value is $\leq \alpha$
- Do not reject H_0 if p-value $> \alpha$

The p-value basically measures the amount of evidence that supports the alternate hypothesis.

We can use p-values to classify sample evidence. In general:

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Example: A commercial aircraft manufacturer buys rivets to use in assembling airliners. Each rivet supplier that wants to sell rivets to the aircraft manufacturer must demonstrate that its rivets meet the required specifications. One of the specs is : "the mean shearing strength of all such rivets is at least 925 pounds." The aircraft manufacturer takes a sample of 50 rivets and discovers that the mean shearing strength of the sample is 919.50. If the standard deviation for the rivets is known to be $\sigma = 18$ lbs is this enough evidence to reject the rivet manufacture's claims about shearing strength with a 5% significance level?

Example (two-tail test) : A coffee dispenser is designed to fill cups with exactly 250ml of liquid. For obvious reasons, it is not good to pour too much or too little coffee into the cups. Assume that the volumes are normally distributed with $\sigma = 14.5$ ml. Is the machine filling properly if 12 cups were filled with an average volume of 255.4ml? Conduct a test of hypothesis at 10% significance level.