The Operating Characteristic Curve

Here we discuss type II errors. The probability of accepting false null hypothesis will be denoted by β.

We can calculate β corresponding to specific alternate values μ.

Recall the coffee dispenser example: A coffee dispenser is designed to fill cups with exactly 250ml of liquid. For obvious reasons, it is not good to pour too much or too little coffee into the cups. Assume that the volumes are normally distributed with $\sigma = 14.5$ ml. We conducted a test of hypothesis at 10% significance level.

We had:

Note: The value of β depends on the acceptance region in the original test and the μ_a (alternate value of the population mean).

The value graph of β vs μ_a is called the **operating characteristic curve (OC-curve)**.

The OC-curve for the coffee dispenser example:

Example: A special battery is supposed to last at least 400 hrs. A hypothesis test will be conducted at 0.02 level of significance. Assume $\sigma = 30$ and $n = 35$. Sketch the OC-curve.

The probability that a test will result in a rejection of a false value for μ is called **the power of the test**. It is equal to $1 - \beta$.

The graph of $1 - \beta$ vs μ_a is called the **power curve**.

Power curve for battery example:

Controlling Both Types of Errors

If sample size n is fixed.

To decrease type I error choose small α but this with increase type II errors.

If we increase sample size we can reduce both types of error.

Example: Determine minimum sample size that will control β at 10% while keeping α at 2% in testing whether batteries have a life of 400hrs. ($\sigma = 30$)

(Assuming we want β for $\mu_a = 385$)