L'Hôpital's Rule (Review)

Note: In what follows a can represent a (finite) real number or ∞ or $-\infty$.

If $\lim_{x \to a} f(x) = 0$ and $\lim_{x \to a} g(x) = 0$ then $\lim_{x \to a} \frac{f(x)}{g(x)}$

is called an independent form of type $0/0$.

Similarly, if $\lim_{x \to a} (x) = \pm\infty$ and $\lim_{x \to a} g(x) = \pm\infty$ then

$\lim_{x \to a} \frac{f(x)}{g(x)}$ is called an independent form of type ∞/∞.

In either case the limit may or may not exist. For limits of this type we can use l'Hôpital's Rule.

L'Hôpital's Rule

Suppose f and g are differentiable functions and that

$$\lim_{x \to a} \frac{f(x)}{g(x)}$$

is an indeterminate form of the type $0/0$ or ∞/∞. Then

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

provided that the limit on the right exists (or is $\pm\infty$).

Examples: Find the following limits (if they exist).

1) $\lim_{x \to 0} \frac{\cos x - 1}{e^x - 1}$

2) $\lim_{x \to 0} \frac{e^x}{1 - \cos x}$

3) $\lim_{x \to \infty} \frac{3 \ln(5x + 3)}{2 \ln(x + 4)}$

Remember, if we don't have and indeterminate form of the type $0/0$ or ∞/∞ then we can't use l'Hôpital's rule. We can, in some cases, rearrange our expression to give us one of these indeterminate forms.

Examples: Find the following limits (if they exist).

1) $\lim_{x \to 0^+} x \ln x$

2) $\lim_{x \to (\pi/2)^-} (\sec x - \tan x)$

Indeterminate Powers

Several indeterminate forms arise from the limit $\lim_{x\to a}[f(x)]^{g(x)}$

1) $\lim_{x\to a} f(x) = 0$ and $\lim_{x\to a} g(x) = 0$ type 0^0

2) $\lim_{x\to a} f(x) = \infty$ and $\lim_{x\to a} g(x) = 0$ type ∞^0

3) $\lim_{x\to a} f(x) = 1$ and $\lim_{x\to a} g(x) = \pm\infty$ type 1^∞

We can evaluate these types of limits using the natural logarithm function and l'Hôpital's rule.

Example:

$$\lim_{x\to 0^+}(1+\sin 4x)^{\cot x}$$