Theorem: **Ratio Test**

Given the series $\sum_{n=k}^{\infty} a_n$

1) If $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = L < 1$ then $\sum_{n=k}^{\infty} a_n$ is absolutely convergent.

2) If $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = L > 1$ or $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \infty$, then the series is divergent.

3) If $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = 1$ then the test is inconclusive.

Examples: Are the following series absolutely convergent, conditionally convergent or divergent?

1) $\sum_{n=0}^{\infty} \frac{3^n}{n!}$

2) $\sum_{n=1}^{\infty} \frac{(2n)!}{n^5}$

3) $\sum_{n=1}^{\infty} \frac{n!}{n \cdot 3^n}$

4) $\sum_{n=0}^{\infty} \frac{(-1)^{n+1}(n+2)}{n(n+1)}$

Theorem: **The Root Test**

Given the series $\sum_{n=k}^{\infty} a_n$

1) If $\lim_{n\to\infty} \sqrt[n]{a_n} = L < 1$ then $\sum_{n=k}^{\infty} a_n$ is absolutely convergent.

2) If $\lim_{n\to\infty} \sqrt[n]{a_n} = L > 1$ or $\lim_{n\to\infty} \sqrt[n]{a_n} = \infty$, then the series is divergent.

3) If $\lim_{n\to\infty} \sqrt[n]{a_n} = 1$ then the test is inconclusive.

Examples: Determine if the following series are convergent, conditionally convergent or absolutely convergent.

1) $\sum_{n=3}^{\infty} \left(\frac{\ln n}{n^2}\right)^n$

2) $\sum_{n=1}^{\infty} \left(\frac{2n+1}{2n+2} \right)^n$

3) $\sum_{n=2}^{\infty} \frac{n^n}{e^n}$