Partial Fractions

In this section we will examine the method of integration for rational functions.

Rational Function - $\dfrac{p(x)}{q(x)}$ where $p(x)$ and $q(x)$ are polynomials.

Common types of integrals

To use the method of partial fractions we will need to be able to integrate the following functions.

1) $\displaystyle\int \frac{1}{(ax+b)^n}dx$

For this type of integral we use a substitution.

Examples: Find the following

a) $\displaystyle\int \frac{1}{3x+2}dx$

b) $\displaystyle\int \frac{-2}{(5x-7)^3}dx$

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2) $\int \frac{ax+b}{(cx^2+dx+g)^n}dx$

Idea: Try to make the numerator look like the derivative of the denominator so that we can use a substitution.

Examples: Find

a) $\int \frac{2x+1}{(2x^2+2x+5)^3}dx$

b) $\int \frac{x-1}{x^2+2x+2}dx$

Again, the goal of this section is to integrate rational functions $\frac{p(x)}{q(x)}$. We want the degree of the numerator to be (strictly) less than the denominator. If it isn't, we perform long division.

Example: Divide the following using long division.

1) $\dfrac{3x^2 + 4x - 3}{x + 2}$

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2) $\dfrac{3x^3 - 5x + 2}{x - 1}$

3) $\dfrac{4x^3 + 8x^2 - x + 6}{x^2 + 1}$

Example: Find the following definite integrals

1) $\int \frac{x^3 + 3x^2}{x^2 + 1} dx$

2) $\int \frac{x^3 + 3x}{x - 1} dx$