The Poisson Distribution

A **Poisson experiment** is characterized by the following properties:

1) The number of successes that occur in any interval is independent of the number of successes that occur in any other interval

2) The probability of a success is the same for all intervals of equal size

Typical Poisson variables:

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Poisson distribution formula

The probability of x success in a Poisson experiment within the specified interval is

$$P(x) = \frac{\mu^x e^{-\mu}}{x!}$$

Where μ is the mean number of successes within the specified interval.

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Example: Suppose that a certain book has an average of 1.5 typos per 100 pages. We randomly select 100 pages.

What is the probability that we find a) no typos, b) one type, or c) two typos?

What is the probability of finding 5 typos within 400 randomly chosen pages of the book?

Poisson distributions are used to approximate binomial probabilities of rare events.

- If the number of trials n in a binomoial experiment is large (greater than 100)
- If $E(x) = \mu = n \cdot p$ is small (less than 10)

then we use the Poisson probability function to estimate the binomial probability.

Example: It is estimated that for any specific Olympic sporting event, one in a thousand spectators will require first aid treatment. What is the probability that 3 spectators out of 2000 will require first aid treatment?

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The Hypergeometric Distribution

The following conditions characterize the **hypergeometric distribution**

1. The population to be sampled consists of N individuals/objects (finite population).
2. Each element in the population can be characterized as a success (S) or a falure (F) with a total of M successes in the population
3. A sample of n individuals is selected randomly without replacement (this means trials are not independent)

Let x = number of successes in a random sample of size n. We know

N = size of population

M = number of successes in population

N-M = number of failures in population

Then the hyper geometric formula is

$$P(x) = \frac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}}$$

Which makes sense since if consider the sample space all samples of size n there are $\binom{N}{n}$ of these. Now the question is, how many of these have x successes in them?

Example: A company employs 30 people, 24 men and 6 women. Five employees are selected at random. What is the probability that there will be 2 women and 3 men in the sample?

Binomial Approximation of the Hypergeometric Distribution

We use a binomial distribution to approximate a hypergeometric distribution if the sample size n is small compared to population N (n is less than 5% of the population, $n \leq 0.05N$). Even though the "trials" (each element sampled) are not independent, the sample size is so small that we assume that they are.

Example: Among 300 income tax returns there are 6 with serious errors. An auditor randomly chooses 5 of the tax returns, what is the probability that 1 of the 5 will have serious errors?

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