More Probability Properties

Proposition: For any event A, $P(A) = 1 - P(A^c)$

Proof:

Proposition: For any events A and B, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Proof:

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Example: In a suburb in Quebec, 60% of households subscribe to an English newspaper, 80% of households subscribe to a French newspaper and 50% of households subscribe both.

If a household is selected at random, what is the probability that a household subscribes to

1) at least one paper.

2) exactly one paper.

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Product Rule (for ordered pairs)

Suppose we have events consisting of ordered pairs and we want to count the number of pairs.

By ordered pairs we mean two-tuples (a,b) where (a,b) is different from (b,a) if $a \neq b$.

If the first element in the pair can be selected in n_1 ways and the second element in n_2 ways then the number of ordered pairs is $n_1 \cdot n_2$.

Note, in order for the product rule to apply there must be the same number of choices for the second element no matter what the choice of first element is.

Example: A family moves to Montreal and needs the services of a dentist and family doctor. There are three family doctors and four dentists accepting new patients. How many possible choices of doctor and dentist can the family choose?

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General Product Rule

Suppose we have ordered collections of k elements $(e_1, e_2, \ldots, e_k)$ and

- there are n_1 for e_1.
- For each choice of e_1 there are n_2 choices for e_2.
- For each choice of e_2 there are n_3 choices for e_3.
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- For each choice of e_{k-1} there are n_k choices for e_k.

Then there are $n_1 \cdot n_2 \cdot \ldots n_k$ k-tuples $(e_1, e_2, \ldots, e_k)$.

Example: Roll a die five times successively noting a sequence of five numbers representing each roll $(R_1, R_2, R_3, R_4, R_5)$. How many possible 5-tuples are there?

Permutations

Any ordered sequence of k objects taken from a set of n objects is called a **permutation of size k.**

The number of permutations of size k from n objects is denoted

$$P_{k,n} \quad \text{or} \quad {}_nP_k$$

Example: How many ways are there to pick a class president and vice-president from a class of 30 students?

Notice that in our example

$$_{30}P_2 = 30 \cdot 29 =$$

This is indeed the formula for $_nP_k$

$$_nP_k = \frac{n!}{(n-k)!}$$

Combinations

Given a set of n distinct objects, any unordered subset of size k that can be formed from the objects is called a **combination**.

The number of combinations of size k from n elements is denoted

$$\binom{n}{k} \quad \text{or} \quad C_{k,n} \quad \text{or} \quad {}_nC_k$$

It makes sense that

$$_nP_k \geq {}_nC_k$$

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Example: Consider the set of objects. {A, B, C, D, E}.

How many ways are there to order k objects?

$$ {}_nC_k = \frac{{}_nP_k}{k!} = \frac{n!}{(n-k)!k!} $$

In our example:

Example: A bridge hand consists of 13 cards selected from a deck of 52.

Consider the events:

A = the event that a hand consists entirely of spades and clubs with both suits represented

B = the event that a hand consists of exactly two suits

Find P(A) and P(B).

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