Recall that events $A_1, A_2, \ldots, A_k$ are mutually exclusive if no two events have any common outcomes. That is

$$A_i \cap A_j = \emptyset, \quad i \neq j$$

Events $A_1, A_2, \ldots, A_k$ are **exhaustive** if one of the events A_i must occur. That is

$$A_1 \cup A_2 \cup \ldots \cup A_k = S$$

The Law of Total Probability

Let $A_1, A_2, \ldots, A_k$ be **mutually exclusive** and **exhaustive** events. Then for any other event B

$$P(B) = P(B|A_1)P(A_1) + P(B|A_2)P(A_2) + \ldots + P(B|A_k)P(A_k)$$

Proof:

Example: For the first test in a stats class

- 60% of the students in the class studied for more than 4 hours
- 28% of the students studied more than 0 hours and less than 4 hours
- 12% of the students did not study for the test

The number of students who passed the test were as follows

- 92% of the students who studied more than or equal to 4 hours passed
- 70% of the students who studied than 0 hours and less than 4 hours passed the test
- 45% of the students who did not study passed the test

What proportion of all the students passed the course?

Baye's Theorem

Let $A_1, A_2, \ldots, A_k$ be k mutually exclusive and exhaustive events with $P(A_i) > 0$ for all i. Then for any other event B for which $P(B) > 0$

$$P(A_j|B) = \frac{P(A_j \cap B)}{P(B)} = \frac{P(B|A_j)P(A_j)}{\sum_{i=1}^{k} P(B|A_i)P(A_i)}$$

Example: 1/1000 people are afflicted with a disease. A diagnostic test has been developed. If a person actually has the disease a positive result will occur 99% of the time. A person without the disease will show a positive result 2% of the time. If a person shows a positive result, what is the probability that they actually have the disease?

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Example. A certain company receives 60 % of their supplies from company X and the rest come from company Y. Company X is late delivering supplies 8 % of the time and company Y is late 5 % of the time. The last delivery of supplies was late. What is the probability that it came from company X ?

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Independence

Two events A and B are **independent** if $P(A|B) = P(A)$

Theorem: If A and B are independent then $P(B) = P(B|A)$

Proof:

Example: Tossing a six-sided die once. Let's define the following events

- A = {2, 4, 6}
- B = {1, 2, 3}
- C = {1, 2, 3, 4}

Are A and B independent? What about A and C?

Proposition: A and B are independent if and only if

$$P(A \cap B) = P(A) \cdot P(B)$$

Proof:

Example: A shipment of 400 grapefruits is sorted below

	Seeds	Seedless
Pink	100	50
Yellow	180	70

Suppose a grapefruit is selected at random. What is the probability that it is

1) Seedless?

2) Pink?

3) Pink and Seedless?

Events $A_1, A_2, \ldots, A_n$ are **mutually independent** if for every combination of events $A_{i_1}, A_{i_2}, \ldots, A_{i_k}$,

$$P(A_{i_1} \cap A_{i_2} \cap \ldots \cap A_{i_k}) = P(A_{i_1}) \cdot P(A_{i_2}) \cdot \ldots \cdot P(A_{i_k})$$

Consider an bag containing four balls, numbered 110, 101, 011 and 000, from which one ball is drawn at random. Let B_k be the event of drawing a ball with 0 in position k. For example B_1 is the event that a ball is drawn with a zero in the first position. Is B_1, B_2, B_3 mutually independent?

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