PROBABILITY EXERCISES

(1) A graduating Dawson student has been accepted at 5 universities, each offering him a choice of 3 degree programmes. If, also, he can take each programme with or without a computer science option, then how many different higher education directions can he take?

(2) Consider a 15 question multiple-choice test. If each question has 4 choices as answers (1 correct, 3 incorrect), then how many different ways can the test be
(a) completed? (b) completed to include no correct answers?

(3) How many lunches consisting of a soup, sandwich, dessert, and a drink are possible if one can select from 4 soups, 3 kinds of sandwiches, 5 desserts, and 4 drinks

(4) Five people are due to speak at a conference. In how many different orders can they
(a) speak? (b) speak, if the keynote speaker must speak last.

(5) Consider the digits $\{1, 2, 3, \cdots, 9\}$. How many different 4-digit numbers can be formed from these digits, if
(a) digits may repeat? (b) digits may not repeat?
(c) digits may not repeat and the numbers must be even?

(6) Consider a pool of 10 Calculus teachers. How many different ways can teachers be assigned (one to a section) in a Calculus course with
(a) 10 sections? (b) 5 sections?

(7) A scrabble player with 7 different letters decides to test all possible 5-letter orderings before playing. If he tests 1 ordering each second, how long will it be before he plays?

(8) How many different ways can 7 men and 2 women sit in a row of 20 seats,
(a) if there are no restrictions?
(b) if the men must sit in the even seats, and the women in the odd seats?

(9) How many different orderings of the letters of the word STATISTICS
(a) are there? (b) begin and end with an S?

(10) How many different orderings of the letters of the word SUCCESSFULLY
(a) are there? (b) are there if the 3 S's must be together in each ordering?
(c) are there, if the word SUCCESS must appear in each ordering?

(11) How many different 7-digit numbers can be formed using the digits of the number 4,221,132?

(12) How many different ways can a 10-question true-false test be
(a) completed? (b) completed to include at least 6 correct answers?

(13) There are 12 points on the circumference of a circle. By joining the points, how many
(a) distinct lines can be formed? (b) distinct triangles can be formed?

(14) In how many ways can 2 different Math. books, 3 different Biology books, and 4 different Psychology books be arranged on a shelf
(a) in any order? (b) if books of the same subject must be together?
(c) if there must be a Math. book at each end?

(15) In how many different teams, consisting of 5 players each, can be chosen from a group of 10 players!

(16) From a group of 5 teachers and 3 students, how many different 4-member committees
(a) can be formed? (b) will include 2 of each?
(c) will include at least 1 of each?

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(17) From a deck of 52 playing cards, how many different 5-card hands
(a) are possible? (b) include only cards of the same suit?
(c) include 3 black and 2 red cards? (d) include at least 1 heart?

(18) A student is selected at random from a class containing 7 Arts students, 12 Science students, and 10 Career students. What is the probability that the selected student is
(a) a Career student? (b) a non-Arts student?

(19) An ordering is selected at random from all the possible orderings of the letters PROBABILITY. What is the probability that the selected ordering
(a) is the one above? (b) begins with P and ends with Y?
(c) has the T in the middle?

(20) A fair coin is tossed 7 times. Find the prob. of observing
(a) only heads. (b) 4 heads and 3 tails. (c) more heads than tails.

(21) A balanced die is rolled 5 times. Find the prob. of observing
(a) only faces less than 3. (b) 2 even and 3 odd faces (c) a total sum of 29.

(22) A student guesses at each of the 10 questions on a multiple choice test. If each question has 3 choices (1 correct, 2 wrong), find the prob. that the student
(a) gets all 10 questions wrong. (b) gets at least 1 question correct.
(c) gets 6 correct and 4 wrong. (d) passes the test ($\geq 60\%$).

(23) A 4-member negotiation committee is to be selected at random from a group of 6 union and 6 management people. What is the prob. that the committee will include (a) equal representation from both sides?
(b) more management than union members?

(24) From a shipment of 20 new TV's, 3 are selected at random for testing. The shipment is only accepted if all 3 test out OK. What is the prob. that the shipment will be accepted, if in fact 2 of the TV's in the shipment are defective

(25) A 5-member committee is selected at random from a group of 6 Conservatives, 7 Liberals and 2 NDP'ers. What is the prob. that the committee will include
(a) only Liberals? (b) 2 Conservatives and 3 Liberals? (c) both NDP'ers?
(d) at least 1 NDP'er? (e) 2 Conservatives, 2 Liberals and 1 NDP'er?

(26) A pair of fair dice is rolled once.
let E = the event of a sum of 6.
let F = the event of a product of 8.
let G = the event of doubles.
(a) Find $P(E)$, $P(F)$, $P(G)$
(b) Find $P(E \cup F)$, $P(E \cup G)$, $P(F \cup G)$
(c) Find $P(E|F)$, $P(F|E)$, $P(E|G)$
$P(G|E)$, $P(F|G)$, $P(G|F)$.

(27) In a certain course, 80% of the students study. Of these students, 75% will pass the course. However, those students who do not study have only a 50% chance of passing the course. What proportion of all the students pass this course?

$P(P|S) = 0.75$
$P(S) = .80$

(28) The chance of a star hockey player playing in his team's next game is only 30%, due to an injury. If he plays, his team is 90% certain to win, otherwise they are equally-likely to win or not. What is the prob. that the team wins its next game?

(29) A certain problem is assigned to each of 3 students. If the students can independently solve this type of problem with probabilities of $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{2}{5}$, respectively, what is the prob. that: (a) all 3 students will solve the problem?
(b) the problem will be solved?

(30) A letter is independently chosen at random from each of the words: CHOICE and CHANCE
What is the prob. that the 2 letters chosen are: (a) both C's? (b) the same?

(31) Consider a family with 6 children. What is the prob. that
(a) at least 1 is a girl? (b) 4 are boys and 2 are girls?
(c) the last child born is the 4th boy?

(32) A trainer claims that his horse has a 70% chance of winning each of the 9 races he is entered in this year. Based on this, what is the prob. that the horse:
(a) will win all 9 races? (b) will lose the first 3 races, and win the last 6 races?
(c) will win 6 races and lose 3 races? (d) will get his 7th win in the last race?

(33) A company reports that there is only a 10% chance that any of its new oil-well drillings will actually yield oil. If this is true, what is the prob. that in its next 15 new drillings
(a) none will yield oil? (b) exactly 5 will yield oil? (c) at least 1 will yield oil?

NOTE: Give only the expressions for the probabilities in the following exercises.

(34) If it is true that $\frac{2}{3}$ rds of all Montréalers are bilingual, then what is the prob. that a random sample of 50 Montréalers will include: (a) exactly 35 who are bilingual?
(b) more than 40 who are bilingual?

(35) Air Canada claims that 5% of all seats sold are "no-shows". Hence, if they sell 104 seats for a 100 seat flight, what is the prob. that they will have "overbooked"?

(36) A meteorologist claims that for any summer day in Montréal, the chance of sunshine is 60%, of clouds, 30%, and of rain, 10%. Hence, if one spends a 2-week summer vacation in Montréal, what is the prob. that one will experience
(a) 14 sunny days? (b) at least 10 sunny days?
(c) 9 sunny days, 4 cloudy days, and 1 rainy day?

(37) At 10 PM on Saturday nights, 30% of the TV viewers in Canada are watching a CBC station, 25% are watching a CTV station, and 45% are watching a USA station. What is the prob. that a poll of 1000 TV viewers taken at that time will include 300 CBC watchers, 250 CTV watchers, and 450 USA watchers?

Probability Exercises — Solutions

(1) $5\times3\times2 = 30$

(2) (a) 4^{15} (b) 3^{15}

(3) $4\times3\times5\times4 = 240$

(4) (a) $5! = 120$ (b) $4! = 24$

(5) (a) $9^4 = 6561$ (b) $9\cdot8\cdot7\cdot6 = 3024$ (c) $8\cdot7\cdot6\cdot4 = 1344$

(6) (a) $10!$ (b) $10\cdot9\cdot8\cdot7\cdot6 = 30{,}240$

(7) $7\cdot6\cdot5\cdot4\cdot3 = 2520$ sec. or 42 min.

(8) (a) $20\cdot19\cdot18\cdot17\cdot16\cdot15\cdot14\cdot13\cdot12$ (b) $(10\cdot9\cdot8\cdot7\cdot6\cdot5\cdot4)\times(10\cdot9)$

(9) (a) $\frac{10!}{3!3!2!} = 50{,}400$ (b) $\frac{8!}{3!2!} = 3{,}360$

(10) (a) $\frac{12!}{3!(2!)^3} = 9{,}979{,}200$ (b) $\frac{10!}{(2!)^3} = 453{,}600$ (c) $\frac{6!}{2!} = 360$

(11) $\frac{7!}{3!2!} = 420$

(12) (a) $2^{10} = 1024$ (b) $\sum_{r=6}^{10}\frac{10!}{r!(10-r)!} = 386$

(13) (a) $\frac{12!}{2!10!} = 66$ (b) $\frac{12!}{3!9!} = 220$

(14) (a) $9! = 362{,}880$ (b) $3!\left[2!\times3!\times4!\right] = 17{,}28$ (c) $2!\left[7!\right] = 10{,}080$

(15) $\frac{10!}{5!5!} = 252$

(16) (a) $\frac{8!}{4!4!} = 70$ (b) $\left(\frac{5!}{2!3!}\right)\times\left(\frac{3!}{2!1!}\right) = 30$ (c) $70 - \frac{5!}{4!1!} = 65$

(17) (a) $\frac{52!}{5!47!} = 2{,}598{,}960$ (b) $4\times\frac{13!}{5!8!} = 5148$ (c) $\left(\frac{26!}{3!23!}\right)\times\left(\frac{26!}{2!24!}\right) = 845{,}000$ (d) $\frac{52!}{5!47!} - \frac{39!}{5!34!} = 2{,}023{,}203$

(18) (a) $\frac{10}{29} = .34$ (b) $\frac{22}{29} = .76$

(19) (a) $\frac{1}{\frac{11!}{2!2!}} = .0000001$ (b) $\frac{\frac{9!}{2!2!}}{\frac{11!}{2!2!}} = .009$ (c) $\frac{\frac{10!}{2!2!}}{\frac{11!}{2!2!}} = .091$

(20) (a) $\frac{1}{2^7} = .0078$ (b) $\frac{\frac{7!}{4!3!}}{2^7} = .2734$ (c) $\frac{\sum_{r=4}^{7}\frac{7!}{r!(7-r)!}}{2^7} = .5000$

(21) (a) $\frac{2^5}{6^5} = .0041$ (b) $\frac{\frac{5!}{2!3!}\times3^2\times3^3}{6^5} = .3125$ (c) $\frac{\frac{5!}{4!1!}}{6^5} = .0006$

(22) (a) $\frac{2^{10}}{2^{10}} = .0173$ (b) $1 - .0173 = .9827$ (c) $\frac{\frac{10!}{6!4!}\times1^6\times2^4}{3^{10}} = .0569$ (d) $\frac{\sum_{r=6}^{10}\frac{10!}{r!(10-r)!}\times1^r\times2^{(10-r)}}{3^{10}} = .0750$

(23) (a) $\frac{\left(\frac{6!}{2!4!}\right)\times\left(\frac{6!}{2!4!}\right)}{\frac{12!}{4!8!}} = .456$

(b) $\frac{\frac{6!}{3!3!}\times\frac{6!}{1!5!} + \frac{6!}{4!2!}\times\frac{6!}{0!6!}}{\frac{12!}{4!8!}} = .273$

(24) $= \frac{\frac{18!}{3!15!}}{\frac{20!}{3!17!}} = 0.716$

(25) (a) $\frac{\frac{7!}{1!2!}}{\frac{15!}{5!10!}} = \frac{21}{3003} = .0067$ (b) $\frac{\left(\frac{6!}{2!4!}\right)\times\left(\frac{7!}{3!4!}\right)}{3003} = .1748$ (c) $\frac{\frac{13!}{7!10!}}{3003} = .0956$ (d) $1 - \frac{\frac{12!}{5!8!}}{3003} = .5714$

(e) $\frac{\left(\frac{6!}{2!4!}\right)+\left(\frac{7!}{2!5!}\right)\times\left(\frac{3!}{1!2!}\right)}{3003} = .2098$

(26) (a) $\frac{5}{36} = .138$, $\frac{3}{36} = .05$, $\frac{6}{36} = .16$

(b) $\frac{5}{36} = .138$, $\frac{10}{36} = .27$, $\frac{8}{36} = .22$

(c) 1, .4, .16, .2, 0, 0

(27) $(.80)\times(.75) + (.20)(.50) = .70$

(28) $(.3)\times(.9) + (.7)\times(.5) = .62$

(29) (a) $\left(\frac{1}{2}\right)\times\left(\frac{3}{4}\right)\times\left(\frac{2}{5}\right) = \frac{3}{20}$

(b) $1 - \left[\frac{1}{2}\cdot\frac{1}{4}\cdot\frac{3}{5}\right] = \frac{37}{40}$

(30) (a) $\frac{2}{6}\times\frac{2}{6} = .11$

(b) $\frac{2}{6}\cdot\frac{2}{6} + \frac{1}{6}\cdot\frac{1}{6} + \frac{1}{6}\cdot\frac{1}{6} = .16$

(31) (a) $1-\left(\frac{1}{2}\right)^6 = .984$ (b) $\frac{6!}{4!2!}\times\left(\frac{1}{2}\right)^6 = .234$ (c) $\left[\frac{5!}{3!2!}\times\left(\frac{1}{2}\right)^5\right]\times\left(\frac{1}{2}\right) = .156$

(32) (a) $(.70)^9 = .040$ (b) $(.30)^3\times(.70)^6 = .003$ (c) $\frac{9!}{6!3!}(.7)^6(.3)^3 = .267$ (d) $\left[\frac{8!}{6!2!}(.7)^6(.3)^2\right]\times(.7) = .208$

(33) (a) $(.90)^{15} = .206$ (b) $\frac{15!}{5!10!}(.10)^5(.90)^{10} = .010$ (c) $1 - .206 = .794$

(34) (a) $\frac{50!}{35!15!}\times\left(\frac{2}{3}\right)^{35}\times\left(\frac{1}{3}\right)^{15}$

(b) $\sum_{r=41}^{50}\frac{50!}{r!(50-r)!}\times\left(\frac{2}{3}\right)^r\left(\frac{1}{3}\right)^{(50-r)}$

(35) $\sum_{r=0}^{3}\frac{104!}{r!(104-r)!}\times(.05)^r\times(.95)^{(104-r)}$

(36) (a) $(.60)^{14} = .0008$

(b) $\sum_{r=10}^{14}\frac{14!}{r!(14-r)!}\times(.60)^r\times(.40)^{(14-r)}$

(c) $\frac{14!}{9!4!1!}\times(.60)^9\times(.30)^4\times(.10)^1$

(37) $\frac{1000!}{300!\,250!\,450!}\times(.30)^{300}\times(.25)^{250}\times(.45)^{450}$